MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 96 No. 7

MEETING OF 1936 MAY 8

Mr. J. H. Reynolds, President, in the Chair.

The President announced that His Majesty King Edward VIII had graciously consented to become the Patron of the Society.

The election by the Council of the following Fellows was duly confirmed :—

Stanley Philip Benge, 84 Muswell Hill Road, London, N. 10 ;
Ralph Judson, Lyndhurst, Lion Road, Bexley Heath, Kent ;
Samuel Keir, F.C.I.S., Rossmore, Scotforth, Lancaster ; and
Arnold Petrie Watson, M.A., 61 Netherhall Gardens, Hampstead, London, N.W. 3.

The President announced that the following had been elected by the Council as Associates of the Society :—

Mr. H. D. Babcock, Mount Wilson Observatory, California ; and
Dr. August Kopff, Astronomisches Rechen-Institut, Berlin.

Seventy presents were announced as having been received since the last meeting, including :—

British Association, Mathematical Tables, vol. v—Factor Table (presented by the B.A. Tables Committee) ;
International Astronomical Union, Transactions, vol. v ; and
U.S. Naval Observatory, Supplement to the American Ephemeris, 1936 —Total Eclipse of the Sun, 1936 June 19 (presented by Dr. Comrie).

The President announced that Dr. A. Kopff would deliver the George Darwin Lecture at the Ordinary Meeting of the Society on Wednesday, June 10, taking as his subject "Star Catalogues, especially those of Fundamental Character."

THE PRESSURE IN THE INTERIOR OF A STAR.

S. Chandrasekhar, Ph.D.

1. In a recent paper * Professor E. A. Milne has established by direct methods certain inequalities which should be true for stellar configurations in hydrostatic equilibrium. A fundamental theorem which is proved in Milne's paper is the following :—

If P_c denotes the central pressure in any equilibrium configuration, and P_1 the pressure at a conventionally assigned boundary where the radius is R and the mass enclosed is M, then

$$P_c > P + \frac{3}{8\pi}\frac{GM^2(r)}{r^4} \geqslant P_1 + \frac{3}{8\pi}\frac{GM^2}{R^4}. \qquad (1)\dagger$$

In the above formula $M(r)$ denotes the mass enclosed inside a sphere of radius r. The inequalities expressed in (1) are true, provided $\bar{\rho}(r)$, the mean density inside r, always exceeds the actual density $\rho(r)$ at r.

2. The outer members in the above inequality yield

$$P_c \geqslant \frac{3}{8\pi}\frac{GM^2}{R^4}. \qquad (2)$$

Formula (2) provides the *minimum* pressure at the centre of an equilibrium configuration of assigned mass and radius. But in certain stellar applica-

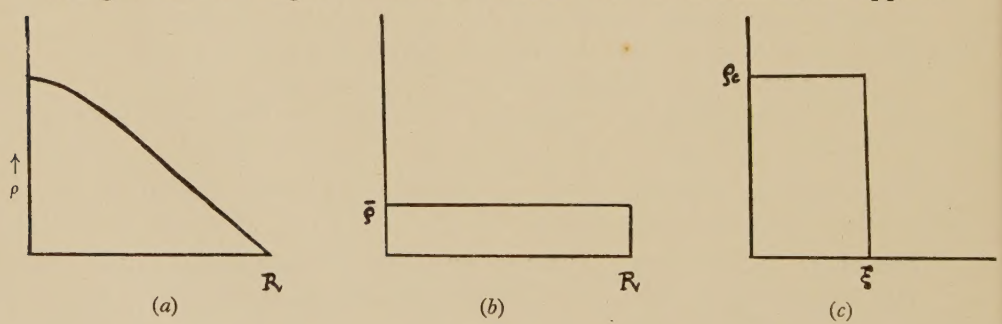

FIG. 1.

tions ‡ the inequality which is required is in the opposite direction to (2), *i.e.* one needs an inequality which would give an upper bound to the central pressure in an equilibrium configuration of assigned mass and central density.

Essentially, inequalities of the kind proved by Milne arise from a "comparison" § of the given equilibrium configuration with another configuration of the same mass and radius but at a uniform density equal to the

* E. A. Milne, *M.N.*, **96**, 179, 1936.

† The outer members of (1) form an inequality which is the statement of a theorem due to Eddington.

‡ E. C. Stoner, *M.N.*, **92**, 662, 1932 ; S. Chandrasekhar, *Observatory*, **59**, 47, 1936.

§ The word "comparison" is not to be taken too literally.

mean density of the original configuration (see fig. 1; compare (*a*) and (*b*)). One gets a different set of inequalities by a corresponding "comparison" of the given configuration with another homogeneous configuration of the same mass but at a uniform density now equal to the central density in the original configuration (see fig. 1; compare (*a*) and (*c*)).

3. *Theorem* 1.—*In any equilibrium configuration in which the mean density inside r decreases outwards we have the inequality*

$$\tfrac{1}{2}G(\tfrac{4}{3}\pi)^{1/3}\bar{\rho}^{4/3}(r)M^{2/3}(r) \leqslant P_c - P \leqslant \tfrac{1}{2}G(\tfrac{4}{3}\pi)^{1/3}\rho_c{}^{4/3}M^{2/3}(r), \tag{3}$$

where $\bar{\rho}(r)$ *denotes the mean density inside r, and* ρ_c *the central density, and* P_c *the central pressure.*

Proof.—The equation of hydrostatic equilibrium is

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho. \tag{4}$$

Integrating this we have

$$P_c - P = G\int_0^r \frac{M(r)}{r^2}\rho dr, \tag{5}$$

or since

$$dM(r) = 4\pi r^2 \rho dr, \tag{6}$$

we have

$$P_c - P = \frac{G}{4\pi}\int_0^r \frac{M(r)dM(r)}{r^4}. \tag{7}$$

By definition

$$\tfrac{4}{3}\pi r^3 \bar{\rho}(r) = M(r). \tag{8}$$

Hence

$$r^4 = \left[\frac{M(r)}{\frac{4}{3}\pi\bar{\rho}(r)}\right]^{4/3}. \tag{9}$$

Substituting (9) in (7) we have

$$P_c - P = \frac{1}{4\pi}(\tfrac{4}{3}\pi)^{4/3}G\int_0^r \bar{\rho}^{4/3}(r)M^{-1/3}(r)dM(r). \tag{10}$$

Since by hypothesis $\bar{\rho}(r)$ decreases outwards we have

$$P_c - P \leqslant \frac{1}{4\pi}(\tfrac{4}{3}\pi)^{4/3}G\rho_c{}^{4/3}\int_0^r M^{-1/3}(r)dM(r), \tag{11}$$

$$\leqslant \tfrac{1}{2}G(\tfrac{4}{3}\pi)^{1/3}\rho_c{}^{4/3}M^{2/3}(r). \tag{12}$$

Again from (10) we have

$$P_c - P \geqslant \frac{1}{4\pi}(\tfrac{4}{3}\pi)^{4/3}G\bar{\rho}^{4/3}(r)\int_0^r M^{-1/3}(r)dM(r), \tag{13}$$

$$\geqslant \tfrac{1}{2}G(\tfrac{4}{3}\pi)^{1/3}\bar{\rho}^{4/3}(r)M^{2/3}(r). \tag{14}$$

Combining (12) and (14) we have the required result.

Corollary 1.—If we put $r = R$ in (12) and (14) we have

$$\tfrac{1}{2}G(\tfrac{4}{3}\pi)^{1/3}\bar{\rho}^{4/3}M^{2/3} \leqslant P_c - P_1 \leqslant \tfrac{1}{2}G(\tfrac{4}{3}\pi)^{1/3}\rho_c{}^{4/3}M^{2/3}, \tag{15}$$

where P_1 is the boundary pressure.

Corollary 2.—If $(1-\beta_c)$ is the ratio of the radiation pressure to the total pressure at the centre of a wholly gaseous configuration satisfying the conditions of theorem 1, then

$$1-\beta_c \leqslant 1-\beta^*, \tag{16}$$

where $(1-\beta^*)$ satisfies the quartic equation

$$M=\left(\frac{6}{\pi}\right)^{1/2}\left[\left(\frac{k}{\mu H}\right)^4\frac{3}{a}\frac{1-\beta^*}{\beta^{*4}}\right]^{1/2}\frac{1}{G^{3/2}}. \tag{17}$$

In terms of $(1-\beta_c)$ the central pressure P_c is given by

$$P_c=\left[\left(\frac{k}{\mu H}\right)^4\frac{3}{a}\frac{1-\beta_c}{\beta_c{}^4}\right]^{1/3}\rho_c{}^{4/3}. \tag{18}$$

Comparing (18) with the inequality on the right-hand side of (15) we have

$$M^{2/3}\geqslant\left(\frac{6}{\pi}\right)^{1/3}\left[\left(\frac{k}{\mu H}\right)^4\frac{3}{a}\frac{1-\beta_c}{\beta_c{}^4}\right]^{1/3}\frac{1}{G}. \tag{19}$$

From (19) the required inequality (16) follows, since $(1-\beta)/\beta^4$ is a montonically increasing function of $1-\beta$.

In the same way one can prove that

$$\frac{1}{G^{3/2}}\left(\frac{6}{\pi}\right)^{1/2}\left[\left(\frac{k}{\mu H}\right)^4\frac{3}{a}\frac{1-\beta_c}{\beta_c{}^4}\right]^{1/2}\geqslant M\cdot\left(\frac{\bar{\rho}}{\rho_c}\right)^2. \tag{19'}$$

4. We shall now establish certain inequalities similar to those in §§ 5, 6 in Milne's paper.

Theorem 2.—*If* I_ν *is the integral defined by*

$$I_\nu=\int_0^R\frac{GM(r)dM(r)}{r^\nu},\qquad(\nu<6), \tag{20}$$

then under the conditions of Theorem 1

$$\frac{3}{6-\nu}\frac{GM^2}{R^\nu}\leqslant I_\nu\leqslant\frac{3}{6-\nu}\frac{GM^2}{\xi^\nu}, \tag{21}$$

where ξ *is defined by the equation*

$$\tfrac{4}{3}\pi\xi^3\rho_c=M. \tag{22}$$

(The first part of the inequality (21) is proved by a different method by Milne (*loc. cit.*, § 5).)

Proof.—We have

$$\begin{aligned}I_\nu&=G\int_0^R\frac{M(r)dM(r)}{r^\nu},\\&=G(\tfrac{4}{3}\pi)^{\nu/3}\int_0^R\bar{\rho}^{\nu/3}(r)M^{(3-\nu)/3}(r)dM(r),\\&\leqslant G(\tfrac{4}{3}\pi\rho_c)^{\nu/3}\int_0^R M^{(3-\nu)/3}(r)dM(r),\\&\leqslant\frac{3}{6-\nu}G(\tfrac{4}{3}\pi\rho_c)^{\nu/3}M^{(6-\nu)/3},\end{aligned} \tag{23}$$

or by (22)

$$I_\nu \leqslant \frac{3}{6-\nu} G \frac{M^2}{\xi^\nu}. \tag{24}$$

From (23) we also have

$$I_\nu \geqslant G(\tfrac{4}{3}\pi\bar{\rho})^{\nu/3} \int_0^R M^{(3-\nu)/3}(r) dM(r) \tag{25}$$

$$\geqslant \frac{3}{6-\nu} G \frac{M^2}{R^\nu}. \tag{26}$$

Combining (25) and (26) we have the required inequality (21).

Corollary 1.—If $\nu = 1$, then I_1 as defined in (20) is precisely the negative potential energy Ω, and (21) now shows that

$$\frac{3}{5}\frac{GM^2}{R} \leqslant \Omega \leqslant \frac{3}{5}\frac{GM^2}{\xi}. \tag{27}$$

Corollary 2.—If $\nu = 2$, then I_2 defines $M\bar{g}$ where $\bar{g}$ is the mean value of gravity in the configuration. By (21) we now have

$$\frac{3}{4}\frac{GM^2}{R^2} \leqslant M\bar{g} \leqslant \frac{3}{4}\frac{GM^2}{\xi^2}. \tag{28}$$

Trinity College, Cambridge :
1936 *April* 28.

THE EQUILIBRIUM OF STELLAR ENVELOPES AND THE CENTRAL CONDENSATIONS OF STARS.

S. Chandrasekhar, Ph.D.

1. In this paper we shall attempt to develop systematically the equilibrium of stellar envelopes. By a stellar envelope we shall mean the outer parts of a star which, though containing only a small fraction (for definiteness we shall assume this fraction to be 10 per cent.) of the total mass M of the star, nevertheless occupy a good fraction of the radius R of the star.

A study of the equilibrium of stellar envelopes has clearly a twofold importance for astrophysical theories. Firstly, it extends the region of the study of stellar atmospheres into the "*far interior*"; and secondly, it has also a very definite bearing on the studies of the "*deep interiors.*" Thus the actual *extent of the envelope* (*defined by the fraction* ξ^* *of the radius* R *which encloses* 90 *per cent. of the mass* M *of the star*) must give some indication of the amount of the concentration of the mass towards the centre of the star under consideration. Thus if ξ^* is very small (say 0·1) then the star must be highly "centrally condensed," while if ξ^* is about 0·5 then there cannot be an excessive concentration of the mass towards the centre—in the latter case ($\xi^* = 0{\cdot}5$) the density distribution is probably close to that of an Emden polytrope of index $n < 3$.

2. The equilibrium of stellar envelopes has been considered earlier by Eddington,* but the point of view in our present paper will be quite different. Eddington assumed that $\xi^* = 0{\cdot}2$, and showed the enhanced opacity discrepancy which would have to be faced by theories of the stellar interior which postulate a high degree of central condensation. On the other hand, one of the chief objects of the present paper will be to calculate on the basis of the *physical theory* of stellar opacity (Strömgren †) the *actual* extents of the envelopes for some typical stars, and to draw from them some general conclusions concerning the dependence of the structure of a star on its position in the Hertzsprung-Russell diagram. Further, we shall see that the formal mathematical theory can be placed on a very much more satisfactory basis than has been attempted so far. Incidentally we are able to get some support to a point of view advocated by Milne ‡ in a rather different connection.

3. In the succeeding paragraphs the formal mathematical theory is developed. The evaluation of the extents of the stellar envelopes for a few typical stars is contained in §§ 12, 16. The rest of the paper is devoted to a general discussion of the results obtained.

4. *The Equations of the Problem.*—In writing down the equations of equilibrium for the stellar envelope we shall introduce two simplifications: (*a*) that there are no sources of energy in the stellar envelope, and (*b*) that the mass contained in the envelope can be neglected in comparison with the mass of the star as a whole. With these simplifications the equations of equilibrium are

$$\frac{d(p+p_r)}{dr} = -\frac{GM}{r^2}\rho, \tag{1}$$

$$\frac{dp_r}{dr} = -\frac{\kappa L}{4\pi c r^2}\rho, \tag{2}$$

where p and p_r denote respectively the gas and the radiation pressures, and κ is the coefficient of stellar opacity, the rest of the symbols having their usual meanings.

The formula for stellar opacity appropriate for our present discussion is that due to Strömgren (*loc. cit.*). According to him we have

$$\kappa = 3{\cdot}89 \times 10^{25}(1 - X^2)\frac{\rho}{T^{3{\cdot}5}}, \tag{3}$$

where X represents the proportion (by weight) of hydrogen in a gram of the stellar material. (There is in addition a "guillotine factor" τ, but we shall neglect its existence for the present—the effects which arise from a consideration of this factor is postponed to a future communication.) We shall write (3) as

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* A. S. Eddington, *M.N.*, **91**, 109, 1930.

† B. Strömgren, *Z. f. Astrophysik*, **7**, 224, 1933, also *ibid.*, **4**, 118, 1932. Also *Handbuch der Astrophysik*, **8**, 124, 1936.

‡ E. A. Milne, *M.N.*, **90**, 17, 1929.

$$\kappa = \frac{\kappa_0 \rho}{T^{3.5}}, \tag{4}$$

where

$$\kappa_0 = 3{\cdot}89 \times 10^{25}(1 - X^2).$$

Dividing (1) by (2) and using (4) we have

$$\frac{dp}{dp_r} = \frac{4\pi cGM}{\kappa_0 L} \frac{T^{3.5}}{\rho} - 1. \tag{5}$$

The equations of equilibrium then are

$$\frac{k}{\mu H} \frac{3}{a} \frac{d(\rho T)}{dT^4} = \frac{4\pi cGM}{\kappa_0 L} \frac{T^{3.5}}{\rho} - 1, \tag{6}$$

$$\frac{a}{3} \frac{dT^4}{dr} = -\frac{\kappa_0 L}{4\pi r^2} \frac{\rho^2}{T^{3.5}}, \tag{7}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho, \tag{8}$$

where k denotes the Boltzmann constant, μ the mean molecular weight, H the mass of the proton and a the Stefan-Boltzmann constant.

The above equations are reduced by the substitutions

$$r = R\xi\,; \qquad T = T_0\tau\,; \qquad \rho = \rho_0\sigma\,; \qquad M(r) = DM\psi \tag{9}$$

(where R and M denote the radius and the mass of the star respectively) to the form

$$K\frac{d(\sigma\tau)}{d\tau^4} = \frac{\tau^{3.5}}{\sigma} - 1, \tag{10}$$

$$\xi^2 \frac{d\tau^4}{d\xi} = -\frac{\sigma^2}{\tau^{3.5}}, \tag{11}$$

$$\frac{d\psi}{d\xi} = \sigma\xi^2, \tag{12}$$

provided

$$\frac{T_0^{3.5}}{\rho_0} = \frac{\kappa_0 L}{4\pi cGM}, \tag{13}$$

$$\frac{T_0^{7.5}}{\rho_0^2} = \frac{\kappa_0 L}{4\pi c} \frac{3}{a} \frac{1}{R}, \tag{14}$$

$$K = \frac{k}{\mu H} \frac{3}{a} \frac{\rho_0}{T_0^3}, \tag{15}$$

$$D = \frac{4\pi R^3 \rho_0}{M}. \tag{16}$$

The solution of the above equations are found to be

$$\rho_0 = \left(\frac{4\pi cGM}{\kappa_0 L}\right)^8 \left(\frac{3}{a}\frac{GM}{R}\right)^7, \tag{17}$$

$$T_0 = \left(\frac{4\pi cGM}{\kappa_0 L}\right)^2 \left(\frac{3}{a}\frac{GM}{R}\right)^2, \tag{18}$$

$$D = \frac{4\pi R^3}{M}\left(\frac{4\pi cGM}{\kappa_0 L}\right)^8 \left(\frac{3}{a}\frac{GM}{R}\right)^7, \tag{19}$$

$$K = \frac{k}{\mu H}\frac{3}{a}\left(\frac{4\pi cGM}{\kappa_0 L}\right)^2 \left(\frac{3}{a}\frac{GM}{R}\right). \tag{20}$$

Numerically the above are equivalent to

$$T_0 = 6{\cdot}41 \times 10^{16}\frac{M^4}{L^2R^2}q^{-2}, \tag{21}$$

$$\rho_0 = 2{\cdot}26 \times 10^{37}\frac{M^{15}}{L^8R^7}q^{-8}, \tag{22}$$

$$K = 2{\cdot}78 \times 10^{9}\frac{M^3}{L^2R}\mu^{-1}q^{-2}, \tag{23}$$

where q is used to denote the "$(1 - X^2)$" which appears in Strömgren's formula for κ. Further, in the above equations M, L and R are expressed in solar units.

5. *The Solution of the Equations.*—Introduce the variable λ defined by

$$\lambda\tau^3 = K\sigma. \tag{24}$$

From (15) we verify that λ as defined above is precisely the ratio of the gas pressure to the radiation pressure, *i.e.* $\beta : (1 - \beta)$ in the usual notation.

In terms of λ equation (10) takes the form

$$\frac{d(\lambda\tau^4)}{d\tau^4} = \frac{K\tau^{1/2}}{\lambda} - 1, \tag{25}$$

which is equivalent to

$$\tfrac{1}{4}\lambda\tau\frac{d\lambda}{d\tau} = K\tau^{1/2} - \lambda(\lambda + 1). \tag{26}$$

Introducing the new variable z defined by

$$K\tau^{1/2} = z, \tag{27}$$

we have the following differential equation for λ:

$$\tfrac{1}{8}z\lambda\frac{d\lambda}{dz} = z - \lambda(\lambda + 1). \tag{28}$$

Instead of (28) consider the more general equation

$$\epsilon z\lambda\frac{d\lambda}{dz} = z - \lambda(\lambda + 1), \tag{29}$$

where ϵ is small positive constant. To solve (29) we shall adopt the method, in principle, due to Jeans.

Assume a solution of the form

$$\lambda = \lambda_0 + \epsilon\lambda_1 + \epsilon^2\lambda_2 + \ldots \tag{30}$$

Inserting the above in (29) and equating the coefficients of the powers of ϵ we get

$$z = \lambda_0(\lambda_0 + 1); \qquad z\lambda_0\frac{d\lambda_0}{dz} = -\lambda_1(2\lambda_0 + 1); \qquad \ldots \tag{31}$$

or

$$\lambda_0 = (z + \tfrac{1}{4})^{1/2} - \tfrac{1}{2}; \qquad \lambda_1 = -\frac{\lambda_0 z}{4(z + \frac{1}{4})}; \qquad \ldots \tag{32}$$

Hence

$$\lambda = \lambda_0\left[1 - \epsilon\frac{z}{4(z + 0{\cdot}25)} + \ldots\right]. \tag{33}$$

The above series converges quite rapidly for small ϵ. Thus for our case where $\epsilon = 1/8$ we have

$$\lambda = \lambda_0\left[1 - \frac{1}{32}\frac{z}{z + 0{\cdot}25} + \ldots\right]. \tag{34}$$

The second term in the brackets in (34) contributes at most about 3 per cent. Hence for our purposes it would be sufficient to use only the first term, in which case

$$\lambda = \lambda_0, \tag{35}$$

or

$$z = \lambda(\lambda + 1). \tag{36}$$

Reverting to our original variable τ we have

$$K\tau^{1/2} = \lambda(\lambda + 1). \tag{37}$$

By (24) then

$$\sigma = \frac{1}{K^7}\lambda^7(\lambda + 1)^6. \tag{38}$$

6. (37) and (38) determine τ and σ in terms of λ. We now proceed to determine the variation of λ with ξ.

From (10) and (11) we have

$$K\frac{\xi^2}{\sigma}\frac{d(\sigma\tau)}{d\xi} = -1 + \frac{\sigma}{\tau^{3{\cdot}5}}, \tag{39}$$

or in terms of λ

$$\frac{\xi^2}{\sigma}\frac{d(\lambda\tau^4)}{d\xi} = -1 + \frac{\lambda}{K\tau^{1/2}}. \tag{40}$$

Using (37) we have

$$\frac{\xi^2}{\sigma}\frac{d(\lambda\tau^4)}{d\xi} = -\frac{\lambda}{\lambda + 1}. \tag{41}$$

Now it is easily verified that

$$\frac{1}{\sigma}\frac{d(\lambda\tau^4)}{d\xi} = \frac{1}{K}\lambda(\lambda+1)(17\lambda+9)\frac{d\lambda}{d\xi}. \tag{42}$$

From (41) and (42) we deduce that

$$(\lambda+1)^2(17\lambda+9)d\lambda = -K\frac{d\xi}{\xi^2}. \tag{43}$$

Integrating the above equation and using the boundary condition that $\lambda = 0$ at $\xi = 1$ we have

$$(\lambda+1)^3(51\lambda+19) - 19 = 12K\left(\frac{1}{\xi} - 1\right). \tag{44}$$

The above equation combined with (37) and (38) determines the physical structure of the stellar envelope.

7. *The Mass-Relation.*—To obtain the mass in the envelope we have to integrate (12). We have

$$\psi(\xi) = \int_\xi^1 \sigma\xi^2 d\xi = \frac{1}{K^7}\int_\xi^1 \lambda^7(\lambda+1)^6\xi^2 d\xi. \tag{45}$$

Using (43) to change the variable of integration to λ we have

$$\psi(\xi) = \frac{1}{K^8}\int_0^\lambda \xi^4\lambda^7(\lambda+1)^8(17\lambda+9)d\lambda, \tag{46}$$

or finally using (44) we have

$$\psi(\lambda) = \frac{1}{K^8}\int_0^\lambda \frac{\lambda^7(\lambda+1)^8(17\lambda+9)d\lambda}{\left[\frac{17}{4K}\{(\lambda+1)^3(\lambda+\frac{19}{51}) - \frac{19}{51}\} + 1\right]^4}. \tag{47}$$

Put

$$x = a\lambda, \tag{48}$$

where

$$a^4 = \frac{17}{4K}. \tag{49}$$

(47) reduces to

$$\psi(\lambda) = \left(\frac{4}{17}\right)^{3\cdot25}\frac{1}{K^{3\cdot75}}f(a\,;\ x), \tag{50}$$

where

$$f(a\,;\ x) = 4\int_0^x \frac{x^7(x+a)^8(x+\frac{9}{17}a)dx}{[(x+a)^3(x+\frac{19}{51}a) - \frac{19}{51}a^4 + 1]^4}. \tag{51}$$

8. *The Equations to Determine the Extent of the Envelope.*—As stated in § 1 we shall define the extent of the stellar envelope by the fraction ξ^* which encloses 90 per cent. of the total mass of the star. By (9) this means that

$$\psi(\xi^*) = \frac{1}{10D}. \tag{52}$$

Let $x = x^*$ where $\xi = \xi^*$. Then by (50)

$$\frac{1}{10} = \left(\frac{4}{17}\right)^{3\cdot25} \frac{D}{K^{3\cdot75}} f(\alpha\,;\ x^*), \tag{53}$$

or using the explicit expressions for K and D given in § 4, we have

$$\frac{1}{10} = \frac{4\pi R^3}{M}\left(\frac{4}{17}\frac{3}{a}\frac{GM}{R}\right)^{3\cdot25}\left(\frac{4\pi cGM}{\kappa_0 L}\right)^{0\cdot5}\left(\frac{\mu H}{k}\frac{a}{3}\right)^{3\cdot75} f(\alpha\,;\ x^*). \tag{53'}$$

Inserting the numerical values for the various quantities occurring in (49) and (53′) and expressing M, L and R in solar units (which convention we shall adopt hereafter) we have finally the following equations which determine the extent of a stellar envelope for any star for which L, M and R are known :—

$$\alpha = 6\cdot25 \times 10^{-3}\left(\frac{L^2 R\mu(1-X^2)^2}{M^3}\right)^{1/4}, \tag{54}$$

$$f(\alpha\,;\ x^*) = 0\cdot0599\frac{(1-X^2)^{1/2}}{\mu^{3\cdot75}}\left(\frac{LR^{0\cdot5}}{M^{5\cdot5}}\right)^{1/2}. \tag{55}$$

$$\xi^* = \frac{1}{(x^*+\alpha)^3(x^*+\frac{19}{51}\alpha)+1-\frac{19}{51}\alpha^4}. \tag{56}$$

(56) is obtained from (44) which in terms of x ($=\alpha\lambda$) has the following form :

$$(x+\alpha)^3(x+\tfrac{19}{51}\alpha)-\tfrac{19}{51}\alpha^4 = \left(\frac{1}{\xi}-1\right). \tag{57}$$

9. *Stellar Envelopes with Negligible Radiation Pressure.*—From (54) we see that α will generally be much less than unity. (For some typical stars (*cf.* § 12) $\alpha \sim 0\cdot05$.) If α can be neglected in comparison with unity then the equations determining the extent of the envelope simplify considerably. From (51) and (57) we then have

$$f(0\,;\ x) = 4\int_0^x \frac{x^{16}dx}{(x^4+1)^4}, \tag{58}$$

$$1+x^4 = \xi^{-1}. \tag{59}$$

Introducing the variable ξ in (58) we have

$$f(0\,;\ \xi) = \int_\xi^1 \left(\frac{1}{\xi}-1\right)^{3\cdot25}\xi^2 d\xi. \tag{60}$$

(60) is precisely what we should have obtained if we had neglected the radiation pressure in the equation of hydrostatic equilibrium. For in that case we have $\tau \propto (\xi^{-1}-1)$ and $\sigma \propto \tau^{3\cdot25}$. (58) is an elementary integral and can be evaluated. The result is

$$f(0\,;\ x) = \frac{585}{96}\left\{x - \frac{1}{4\sqrt{2}}\left[\log\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1} + 2\tan^{-1}\frac{x\sqrt{2}}{1-x^2}\right]\right\}$$
$$-\frac{x^{13}}{3(x^4+1)^3} - \frac{13x^9}{24(x^4+1)^2} - \frac{117x^5}{96(x^4+1)}. \tag{61}$$

The function $f(o\,;\,\xi)$ is tabulated in Table I.

TABLE I

$f(o\,;\,\xi)$

ξ	$f(o\,;\,\xi)$	ξ	$f(o\,;\,\xi)$
0·85	0·0000	0·50	0·0242
·80	·0004	·45	·0398
·75	·0010	·40	·0642
·70	·0021	·35	·1015
·65	·0042	·30	·1587
·60	·0080	·25	·2477
·55	·0142	·20	·3905

TABLE II

x	$a=0·05$		$a=0·15$		$a=0·25$	
	ξ	f	ξ	f	ξ	f
0·40			·930	0·000002	0·882	0·000002
·45			·902	·00001	·844	·000009
·50			·868	·00003	·801	·000034
·55			·828	·00009	·753	·000105
·60			·783	·00023	·702	·000287
·65	0·813	0·0002	·735	·00053	·649	·00070
·70	·767	·0005	·683	·00110	·596	·00154
·75	·718	·0011	·630	·00206	·543	·00311
·80	·666	·0025	·577	·00356	·492	·00578
·85	·612	·0050	·525	·00571	·444	·01005
·90	·559	·0095	·475	·00860	·399	·01643
·95	·508	·0169	·428	·01225	·357	·02548
1·00	·459	·0281	·384	·01663	·319	·03772
1·05	·413	·0441	·344	·02164		
1·10	·370	·0661				
1·15	·331	·0948				
1·20	·296	·1311				

10. If a cannot be neglected in comparison with unity recourse must be made to numerical methods. It is convenient to tabulate $f(a\,;\,x)$ for certain specified values of a. The numerical integration has been effected for the three cases $a=0·05$, 0·15 and 0·25, and the results are tabulated in Table II.

11. *Choice of the Molecular Weight* μ.—Before we can proceed to make use of the equations (54), (55) and (56) to determine the actual extents of

the stellar envelopes for some typical stars, it is first necessary to decide upon the suitable values for μ and X appropriate to each individual star. This is important, inasmuch as in the principal equation determining the extent of the envelope (namely (55)) the molecular weight μ occurs with a high power. In our calculations we shall use the values of μ (and X) which have been computed by Strömgren. Now the use of Strömgren's μ's and X's may at first be regarded as a drawback in this connection. For, one of the main objects of computing the extents ξ^* for individual stars is to get some *direct* information on the complete march of the density distribution in the star as a whole, and Strömgren's μ's and X's were computed on the basis of a particular theory of the stellar interior and an assumed model for the density distribution. Nevertheless the use of Strömgren's μ's and X's for a general guidance can be justified on the ground that on the basis of his *derived* values for μ Strömgren is able to obtain a reasonable quasi-empirical interpretation of the Hertzsprung-Russell diagram. It might be recalled in this connection that his interpretation was based on the discovery that in conformity with the Russell-Vogt theorem the stars can in fact be arranged in a two parametric family, the two parameters being M and (the derived) μ. If μ varies from star to star—we shall obtain some further evidence for this in this paper (*cf.* § 18)—then it is reasonable to expect that there are systematic variations of μ in the plane of the Hertzsprung-Russell diagram. Of course there is no *a priori* reason why the variations should be of the type discovered by Strömgren, but strong empirical evidence favouring Strömgren's picture has recently been obtained by Dr. Kuiper who first noticed the very remarkable similarity of the *actual* Hertzsprung-Russell diagrams for clusters and the Strömgren curves of constant μ in the Hertzsprung-Russell plane.

Finally the extents computed on the basis of Strömgren's μ's and X's can in one sense be regarded as indicating the *maximum* central condensations inasmuch as if $\xi^*(\mu_1)$ and $\xi^*(\mu_2)$ are the calculated extents for the envelope of the same star but for two different values of μ, namely μ_1 and μ_2, then

$$\xi^*(\mu_1) > \xi^*(\mu_2) \quad \text{if} \quad \mu_1 > \mu_2. \qquad (62)^*$$

For the above reasons we shall use Strömgren's μ's and X's for the numerical evaluation of the extents for some typical stars, to which we proceed now.

12. *The Extents of the Stellar Envelopes for some Typical Stars.*—The procedure to calculate the extent of the envelope of a given star of known mass, radius and luminosity (and an assumed molecular weight μ) from the equations (54), (55) and (56) is of course obvious. We have first to calculate α according to (54) and (55) then determines the value of x^* such that $f(\alpha\,;\,x^*)$ has the value given by the right-hand side of (55). Finally (56) determines the corresponding value ξ^*. In practice it is, however, more convenient to plot $f(\alpha\,;\,x)$ against ξ and to read off the value of ξ for which f takes the required value. For most of the stars for which the extents were calculated α was near 0·05, and graphical interpolation both across

* It might be pointed out in this connection that as we have neglected the "guillotine factor" (see § 4) the ξ^* calculated, even so, are slightly underestimates.

and down the figures of Tables I and II sufficed to determine ξ^* fairly accurately.

The observational data concerning the masses, radii and luminosities have been collected together by Strömgren * in his Table IV. His derived μ's and X's are given in his Table V. The information provided by these tables is sufficient to calculate the corresponding extents for the envelopes of the respective stars. The results of the calculations are summarised in Table III below. The values of μ and X are taken from Strömgren. The last column gives the extents.

13. *Dependence of the Structure of a Star with its Position in the Hertzsprung-Russell Diagram.*—The most striking feature of the extents given in the last column of Table III is the surprising constancy of ξ^* for the main series stars up to spectral type ~B8. For these stars ξ^* is about 0·43. This constancy is all the more striking as we have allowed for the variation of μ according to Strömgren.

The only giants in the list are Capella A and B, and if one can judge from these, then it would appear that the giants on the whole have much less central condensation than the main series stars. One could have anticipated this result from Strömgren's interpretation of the Hertzsprung-Russell diagram. On his ideas, if we consider a giant (or a super-giant) and also a main series star of about the same absolute bolometric magnitude, then the giant should be relatively poorer in hydrogen (as compared with the main series star), and this would (according to our equations for the envelope) predict a greater value ξ^* for the giant than for the main series star, *i.e.* the giants should be expected to be more "homogeneous" than the main series stars.

Another result indicated by the table is that the massive B stars (on the main series) have much less central condensation than the less massive stars further down the series. The only reliable examples we have are the V Pup and Y Cyg systems.† Actually the extents are $\xi^* = 0{\cdot}55$ and $\xi^* = 0{\cdot}51$ respectively, and it has to be especially noticed that these values are obtained for $\mu = 0{\cdot}6$—a higher value for μ would give much larger values for ξ^*. It should further be pointed out in this connection that for an increase of ξ^* from 0·43 to 0·51 the corresponding value of f should have to be smaller by about a factor 3.

14. The general conclusions then are

(*a*) The main series stars up to spectral type ~B8 are characterised by a more or less constant $\xi^* \sim 0{\cdot}43$.

(*b*) Higher up the main series there is a definite tendency for ξ^* to increase, *i.e.* in this region the stars tend to become more "homogeneous."

(*c*) The giants are characterised by greater values for ξ^* than the corresponding stars on the main series with the same luminosities.

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* *Z. f. Astrophysik*, **7**, 222, 1933 (referred to as *loc. cit.*).

† The two components of the V Pup and Y Cyg systems seem to be "identical," and consequently in our calculations the data were combined to give a "mean" V Pup $\frac{1}{2}(br+f)$ and a mean Y Cyg $\frac{1}{2}(br+f)$.

TABLE III

Star	Mass/⊙	μ	X	α	f	ξ^*
Capella A	4·18	1·01	0·30	0·044	0·0235	0·48
Capella B	3·32	1·00	·31	·032	·0274	·48
Sirius A	2·45	0·90	·40	·0195	·0472	·415
α Cen A	1·10	0·96	·37	·0061	·0569	·411
α Cen B	0·94	1·10	·28	·0057	·0535	·417
The Sun	1	0·98	·37	·0063	·0601	·405
V Pup *br* / V Pup *f*	18·5	0·6	·8	·049	·0105	·55
Y Cyg *br* / Y Cyg *f*	17·2	0·6	·8	·0625	·0156	·51
u Her *br*	10·0	0·6	·7	·049	·0355	·44
u Her *f*	3·4	1·23	·19	·050	·0302	·45
σ Aql *br*	6·2	0·86	·42	·068	·0313	·44
σ Aql *f*	5·1	0·89	·40	·055	·0319	·44
β Per *br*	4·7	0·75	·53	·030	·0416	·43
U Oph *br*	5·4	0·79	·49	·044	·0367	·44
U Oph *f*	4·7	0·85	·43	·042	·0352	·44
Z Vul *br*	7·6	0·6	·7	·037	·0476	·413
Z Vul *f*	3·0	1·05	·27	·027	·0293	·47
β Aur *br* / β Aur *f*	2·38	1·185	·205	·044	·0380	·43
WW Aur *br*	2·19	1·00	·32	·021	·0428	·43
WW Aur *br*	1·90	1·06	·28	·021	·0445	·43
RX Her *br*	2·09	1·12	·24	·030	·0419	·43
RX Her *f*	1·86	1·14	·24	·027	·0431	·43
TX Her *br*	2·06	0·96	·35	·0185	·0492	·415
TX Her *f*	1·77	0·97	·34	·0138	·0476	·415
TV Cas *br*	2·4	1·20	·20	·046	·0366	·435
TV Cas *f*	1·2	1·56	·07	·0295	·0339	·45
Z Her *br*	1·6	1·00	·32	·013	·0485	·42
Z Her *f*	1·3	1·34	·15	·018	·0318	·47

15. An immediate consequence of the conclusions stated in § 14 is that the Cepheids and the Cluster type variables which occur in the "supergiant" region of the Hertzsprung-Russell diagram must be much less condensed towards the centre than the typical main series stars. The necessity for the Cepheids to be more homogeneous than "ordinary stars" has often been emphasised by Eddington.

16. *The Structure of the Trumpler Stars.*—In § 13 we saw that the two reliably known massive stars, namely, V Puppis and Y Cyg, are both characterised by substantially larger values for ξ^*. From this we concluded the possibility that still higher up the main series the stars are probably characterised by still larger values for ξ^*. This prediction can now be confirmed by computing the ξ^*'s for the very massive stars recently discovered by Trumpler in open clusters.* The uncertainty in his derived

Table IV

Star	Mass/⊙	a	f	ξ^*
N.G.C. 2244 15	49	0·18	·0055	0·51
N.G.C. 7380 1	59	·08	·0016	$>$ ·65
N.G.C. 2244 8	74	·05	·00065	$\sim$ ·75
N.G.C. 2264 60	120	·08	·00042	$>$ ·7
N.G.C. 6871 2	170	·065	·00017	$>$ ·8
N.G.C. 2362 1	230	·08	·00007	$>$ ·8
N.G.C. 6871 5	286	·03	·00003	$>$ ·85

masses and radii is probably due only to the uncertainty in the temperature scale, but as Dr. Kuiper has pointed out to the writer this uncertainty cannot, however, affect Trumpler's values for the masses of his stars by a factor less than 0·75. With the revised data provided by Dr. Kuiper the extents ξ^* were calculated for the Trumpler stars assuming $\mu = 0{\cdot}6$ and $X = 0{\cdot}8$. The results are given in Table IV.

In considering the values of ξ^* given in Table IV it has to be remembered that they are the *minimum values*, since μ was taken to be equal to 0·6. If the observational data are at all reliable † the conclusion must be that the Trumpler stars are very much more homogeneous than any of the other known stars.§ The results in Table III already indicate a tendency for ξ^* to increase higher up the main series, and our present calculations confirm that view.

17. *The Problem of the Massive Stars.*—The conclusion reached about the structure of the massive stars in §§ 13, 14, 16 have a bearing on the general problem of the massive stars. It was shown by the author,‡ and

* R. J. Trumpler, *Pub. Astro. Soc. Pac.*, **47**, 254, 1935.

† It would be sufficient if they are even only qualitatively reliable.

‡ *Zeits. f. Astrophysik*, **5**, 321, 1932.

§ *Cf.* A. Beer and S. Chandrasekhar, *Observatory*, **59**, 168, 1936.

elaborated to considerable length since,* that the generally accepted formula for relativistic degeneracy † leads unambiguously to the result that the massive stars cannot become white dwarfs. This arises because there is firstly an upper limit M_3 to the mass of a completely degenerate configuration, and secondly for a star of mass greater than

$$\mathfrak{M} = M_3 \beta_\omega^{-3/2}, \tag{63}$$‡

where β_ω is such that

$$\frac{960}{\pi^4} \frac{1-\beta_\omega}{\beta_\omega} = 1, \tag{64}$$

degeneracy is not possible. This result contradicted the then current view that any star of any assigned mass "must" eventually become a white dwarf. The writer then proposed the view that if a massive star *has* to become a static white dwarf then the only way for it would be to get rid of its excess mass. It was further thought possible that the observed Wolf-Rayet phenomenon of the radial ejection of matter may have a bearing on this problem. This question has recently been discussed from a different point of view by ten Bruggencate.§

A further possibility is now offered by our present conclusions concerning the structure of the massive stars and also the circumstance to which we have drawn attention to, namely, that the giants are characterised by larger values for ξ^* than the corresponding main series stars. Now Strömgren has already pointed out that the track of evolution of a star in the Hertzsprung-Russell diagram will be along a curve of constant mass in the direction of increasing radius and diminishing hydrogen content. The very massive stars *could* therefore be expected to travel right across the Hertzsprung-Russell diagram and end up as a nebulous mass. This suggestion was first

* *M.N.*, **95**, 207, 226, 676, 1935. For a concise summary on these matters see B. Strömgren, *Handbuch der Astrophysik*, **8**, 156–168, 1936.

† Eddington has published a note (*M.N.*, **96**, 20, 1935) criticising the proof of the relativistic degeneracy formula which Dr. Møller and the writer have published (*M.N.*, **95**, 673, 1935). We are, however, unable to accept Eddington's criticisms. In particular, his appeal to the uncertainty principle seems to us fallacious. Actually the discrete eigen values for p were obtained using a periodicity condition (bottom of page 674) and consequently there can be no contradiction with the uncertainty principle. We clearly do not know which of the electrons are in which of the "compartments." Eddington's second criticism (top of page 21) arises from his having overlooked the fact that we normalised the wave function per unit volume. The appearance of the volume factor V in our equation (12) makes this explicit. The writer shares the view of most theoretical physicists that the generally accepted formula for relativistic degeneracy is *the* one which is consistent with the general scheme of quantum mechanics.

‡ That $\mathfrak{M}$ as defined in (63) and (64) represents the maximum mass of a star which could consistently be regarded as wholly "degenerate" is easily proved. Consider a completely degenerate configuration of mass slightly less than M_3. The density is everywhere so great that we could increase the radiation pressure from zero to a value only slightly less than $(1-\beta_\omega)$ at *each* point of the configuration and still regard the matter as degenerate. The mass of the new equilibrium configuration so obtained is approximately $M\beta_\omega^{-3/2}$. When $M \to M_3$, the result becomes exact.

§ P. ten Bruggencate, *Z. f. Astrophysik*, **11**, 201, 1936.

made to the writer by Dr. Kuiper. Our present calculations on ξ^* lend support to Dr. Kuiper's suggestion inasmuch as, firstly, the massive stars are characterised by substantially larger values for ξ^*, and secondly, if the stars evolve in the direction of increasing μ and increasing radius they would be expected to become still more "homogeneous." Kuiper's suggestion, therefore, that massive stars might eventually end up as nebulous masses (in contrast to stars of mass less than M_3 which become white dwarfs) seems to the writer as the most probable of the suggestions that has so far been made concerning the evolution of the massive stars.

18. *Concluding Remarks.*—We have seen how from the observed masses, luminosities and radii of stars, it is possible to infer something about the internal density distributions in the stars. The extent ξ^*, which in a sense is a measure of the amount of the concentration of the mass towards the centre, is a function of the mass, radius, luminosity and the mean molecular weight μ. Conversely if we had assumed that ξ^* and μ are the same for all stars then we should have obtained a *relation* between M, L and R. Before the systematic variation of μ in the plane of the Hertzsprung-Russell diagram was discovered by Strömgren, one had always assumed that μ is about the same for all stars. Consequently with an assumed constant value for μ the hypothesis that "*all stars are built alike*" (ξ^* is the same for all stars) would be sufficient to derive a relation between L, M and R. It is not necessary to solve the problem of the equilibrium of a star as a whole to derive a relation between mass radius and luminosity. It is precisely this last point which was emphasised by Milne in his 1929 paper.* Of course our present background mathematical analysis is quite different from Milne's, but it is satisfactory to get a confirmation of his fundamental point, namely, that a specific combination of the observed L, M and R should give us some information concerning the internal density distribution in the star without any special theory for the deep interior.

From our present point of view where μ is assumed to vary from star to star we *could* have calculated μ for individual stars assuming that ξ^* was to be the same for all stars. We have not, however, preferred to do this for the reasons stated in § 11. It is therefore highly satisfactory that ξ^* does show a surprising constancy for the main series stars up to spectral type ~B8. But we have also shown that ξ^* is *not* the same for *all* stars, since even with $\mu = 0{\cdot}6$ the massive stars have substantially larger values for ξ^*.

Summary.—In this paper the equilibrium of stellar envelopes is considered and methods developed to compute the fraction ξ^* of the radius of a star which encloses 90 per cent. of its mass. The main results of such calculations are summarised in Tables III and IV. The general conclusions which follow are stated in § 14. § 17 contains a discussion of the general problem of the massive stars raised by the author's earlier studies on the stellar configurations with degenerate cores.

Trinity College, Cambridge :
1936 *May* 2.

* *M.N.*, **90**, 17, 1929.

ON THE INTERPRETATION OF INTERSTELLAR LINES.

C. S. Beals.

1. Previous measures of the intensities of interstellar lines * have indicated that the equivalent widths in kilometres per second of the two members of the sodium doublet λλ 5889·96, 5895·92 are equal to or slightly greater than those of the corresponding members of the ionized calcium doublet λλ 3933·66, 3968·46. This result is not in accord with former theoretical estimates of relative intensity by Sir Arthur Eddington † and by Gerasimovič and Struve.‡ They agreed in finding that neutral sodium atoms in interstellar space were outnumbered several hundred times by atoms of ionized calcium. Moreover, they also agreed, on the basis of the theoretical estimates then available, that the atomic absorption coefficient of Ca^+ was at least as great as that of *Na*. The appearance of interstellar sodium lines with intensities approximately equal to those of calcium accordingly appeared very difficult to explain.

Recently Sir Arthur Eddington has put forward a new suggestion to account for the approximate equality of the sodium and calcium lines. According to this suggestion the differential velocity due to galactic rotation is the agency mainly responsible in forming the line profile. Neglecting wings due to the form of the atomic absorption coefficient and temperature motions of the atoms he has shown that an interstellar line appearing in the spectrum of a given star will, to a first approximation, have a rectangular profile with a width in km./sec. equal to the component of the radial velocity of the star due to galactic rotation.§ Since this theory is based on the assumption that the motion of galactic rotation is shared alike by the stars and the atoms of interstellar matter, the lines due to sodium and calcium will have the same width and the same rectangular form and can differ only in depth. If the density of the less abundant atom (presumed to be sodium) be sufficient to cause complete absorption over the width of the line, then, in spite of a difference in density corresponding to a factor of 300, the lines will be of approximately equal intensity.

Almost beyond doubt the mechanism outlined above may be expected to play a dominant rôle for distant stars located in regions of the sky where the galactic rotation effect in radial velocity is reasonably large. For many of the nearer stars, however, the matter is less certain and there remain aspects of the observational data which are difficult to explain on this basis. In this paper attention is directed to some of these apparent discrepancies, and the suggestion is advanced that for many, if not a majority, of the stars

* *M.N.*, **93**, 585, 1933 ; **94**, 663, 1934.
† *Proc. Roy. Soc.*, A, **111**, 424, 1926 ; *Internal Constitution of the Stars*, 1926.
‡ *Ap. J.*, **69**, 7, 1929.
§ Or twice the interstellar velocity.

so far observed, irregular or peculiar motions within the cloud may assume an importance equal to that of galactic rotation in forming the line profile. The density of the absorbing material is also touched upon, and while no numerical calculation is attempted evidence is presented which suggests that the density may be considerably less than had formerly been supposed. Finally, the relative intensities of sodium and calcium lines are re-examined with the aid of new material, and the writer's previous conclusions, indicating greater strength for the sodium lines, are confirmed.

2. *Observational Material.*—The observational data on which this discussion is based have been mainly obtained at the observatory, though some use has been made of measures published by Williams.* The D.A.O. material consists of 55 stars, and 330 plates have been measured, an average of 6 plates per star. A number of the intensities have already been published, though in most cases the accuracy has been improved by the addition of new observations. The observations have been mainly confined to stars of type Bo or earlier in order to eliminate as far as possible effects due to blends with stellar lines.

General spectrophotometric methods have been described in previous publications.† Recent changes have included the conversion of the direct reading microphotometer previously described into one of self-recording type, and the replacement of the neutral tint wedge by a rapidly rotating step sector (100 revs. per sec.), as described by Webb ‡ for purposes of plate calibration. Tests by the writer and by other members of the staff have indicated that these changes have had no observable effect on measures of intensity.

Total intensities, now ordinarily referred to as equivalent widths, have been derived by plotting individual values of the intensities across the line profile and measuring with a planimeter the area enclosed between the profile and the continuous spectrum. The dispersion co-ordinate employed is the Doppler displacement in kilometres per second. In most cases the profile is mainly due to the properties of the instrument, although in a few stars the interstellar lines apparently have a real width great enough to be detected with the dispersion available. In many cases the area on the plate occupied by the line is of the order of 0·015 sq. mm., so that with the fast plates ordinarily employed a large scatter in the individual measures is unavoidable.

The measures of intensity for all but a few of the brighter stars have been made with the spectrograph in its one-prism form, using cameras of 42 cm. and 71 cm. focal length with dispersions of 33 A. and 20 A. per mm. at K respectively. Most of the earlier measures were made on plates taken with the shorter focus camera, while recently the 71 cm. instrument has been almost exclusively used. In Table I results obtained with the two different cameras are compared. In spite of individual scatter the general agreement is reasonably good, and the absence of any very appreciable

* *Ap. J.*, **79**, 280, 1934.
† *Pub. D.A.O.*, **6**, 95, 1934; *M.N.*, **94**, 663, 1934.
‡ *Journ. Opt. Soc. America*, **23**, 157, 1933.

systematic effect affords some grounds for confidence in the reliability of the measures.

TABLE I

Comparison of Intensities obtained with 42 cm. and 71 cm. Cameras

Star	Int. K, 71 cm. Camera	No. Plates	Int. K, 42 cm. Camera	No. Plates	Ratio 71/42
H.D. 108	34	3	40	4	·85
H.D. 1337	41	2	32	2	1·28
H.D. 47839	15	1	16	1	·94
H.D. 190864	51	2	66	1	·77
H.D. 191765	33	1	39	4	·85
H.D. 192163	33	1	36	4	·92
H.D. 192639	25	1	23	2	1·09
H.D. 193237	33	2	35	6	·95
H.D. 193793	51	3	46	1	1·11
H.D. 198846	27	1	31	2	·87
H.D. 203064	22	1	26	6	·85
H.D. 209975	20	2	21	2	1·05
H.D. 210839	22	4	26	8	·85
H.D. 218915	48	1	34	3	1·40
				Mean	·99

In Table II the intensities of lines measured in common by Williams and the writer are compared. Again the agreement on the whole is reasonably good though relatively large differences are found for individual stars. With the exception of three stars, indicated in the table by asterisks, the measures were made with comparable dispersion, 16·6 and 23·5 A. per mm. being used by Williams and 33 and 20 A. per mm. by the writer.

While a majority of the measures of Ca^+ lines have been made with single-prism dispersion as described above, a number of the brighter stars have been observed with the spectrograph in its three-prism form. The cameras employed were of 71 cm. and 96 cm. focal length, giving dispersions at K of 6·5 and 4·5 A. per mm. In Table III are shown the results of measures made with widely differing dispersion. Most of the data in this table are self-explanatory. The letters W and B in the last column refer to observations by Williams and the writer respectively. The figure 16·6 A. per mm. has been quoted for the dispersion used by Williams, though it may be that the smaller dispersion, 23·5 A. per mm., has been used for some of the measures.

The general tendency shown by the measures of Table III is to indicate that larger values of intensity are associated with smaller dispersion and *vice versa*. Of particular interest in this connection are the values of K intensity for Nova Herculis, which differ by a factor of 2. It is possible that the differences in this case are due to changes in the intensity of the line due to blends, but the weight of evidence is against this view. There

TABLE II

Comparison of Intensities obtained by Williams and Beals

Star	Int. K, Williams	No. Plates	Int. K, Beals	No. Plates	Ratio Williams/Beals
H.D. 13854	54	1	61	5	0·89
H.D. 14134	53	1	69	4	0·77
H.D. 24912	16	1	14	13	1·10
H.D. 30614	29	2	29	5	1·00
H.D. 36861	18	2	17	5	1·06
H.D. 37128	13	1	9*	4	1·44
H.D. 47839	19	4	16	7	1·19
H.D. 91316	14	3	19*†	10	0·74
H.D. 149757	6	2	6*	3	1·00
H.D. 149881	25	1	28	5	0·89
N. Herculis	14	15	13	5	1·08
H.D. 193237	28	2	34	8	0·82
H.D. 193322	40	1	36	1	1·11
H.D. 210839	27	1	26	12	1·04
H.D. 214680	11	2	18	1	0·61
				Mean	·98

* Dispersion 6·5 A. per mm.
† Blend of stellar and interstellar line.

seems to be no obvious theoretical reason why a measure of total absorption should depend on the dispersion, but in actual practice, as has already been pointed out by Minnaert,* observers have usually found that larger values of total absorption result when smaller dispersion is employed. The effect seems to be greatest for faint lines at the limit of visibility and to decrease for lines of greater intensity. Owing to the difficulty of making reliable corrections for this effect, comparable dispersion has been employed wherever possible when measures of intensity are compared. For this reason the absolute intensities quoted in Tables VII, VIII and XI are not always consistent, the discrepancies being due to the necessity of selecting the dispersion which has most significance for the particular comparison which is being made.

Most of the plates of the sodium D lines were secured with the use of three prisms in conjunction with the 71-cm. camera. The resulting dispersion, 30 A. per mm., is of the same general order as that ordinarily used in the violet. It is interesting to note that if the dispersion be reckoned in terms of Doppler displacements in km./sec. the 71-cm. camera used with one prism at K gives a dispersion of 1518 km./sec. per mm., while the same camera used with three prisms gives for D2 a dispersion of 1554 km./sec. per mm. It is hoped that this similarity in dispersion combined with the

* *Observatory*, 57, 328, 1934.

TABLE III

Comparison of Intensities obtained using widely Differing Dispersion

Star	H.D.	λ	No. Plates	Disp. A. per mm.	Int. km./sec.	Observer
o Per	23180	3933	3	20	10·3	B
,,	,,	,,	1	9	6·0	B
,,	,,	,,	1	6·8	7·0	B
,,	,,	,,	1	6·5	8·6	B
ξ Per	24912	3933	14	33	14·0	B
,,	,,	,,	1	16·6	16·0	W
,,	,,	,,	4	4·5	9·1	B
S Mon	47839	3933	4	33	16	B
,,	,,	,,	1	20	15	B
,,	,,	,,	2	16·6	19	W
,,	,,	,,	1	13·5	14	B
,,	,,	,,	1	4·5	14	B
ε Ori	37128	3933	2	16·6	13	W
,,	,,	,,	4	6·5	9·1	B
ι Ori	37043	3933	2	20	4·4	B
,,	,,	,,	4	6·5	6·2	B
ζ Oph	149757	3933	1	16·6	6·0	W
,,	,,	,,	3	6·5	6·0	B
N Her		3933	5	20	13·4*	B
,,		,,	15	16·6	14·0†	W
,,		,,	7	4·5	7·4‡	B
P Cyg	193237	5889	3	66	29	B
,,	,,	,,	4	22	24	B
,,	,,	5895	3	66	25	B
,,	,,	,,	4	22	27	B
λ Cep	210839	3933	10	33	26	B
,,	,,	,,	1	6·5	21	B

* Plates taken 1935 February 2.
† ,, ,, 1935 January 22–March 26.
‡ ,, ,, 1935 February 8–February 28.

similarity of the slit pattern, due to the use of the same camera in both regions, will tend to minimise systematic errors in the comparisons of sodium and calcium lines. A few plates in the D region have been taken with a camera of 96 cm. focal length, giving a dispersion of 22 A. per mm., while 3 plates of the D lines of P Cygni were secured with the 96 cm. camera used with one prism. The dispersion of this combination is 66 A. per mm.

For a few stars of low declination and in particular for the star ζ Ophiuchi with a declination of −10° the well-known "rain band" due to water vapour, having approximately 150 lines in the region λ 5850–6000,* has sometimes occasioned difficulty in the measurement of the sodium lines. Corrections for this effect have been worked out by observing the water-vapour band with the stellar spectrograph, using stellar and terrestrial sources in which the sodium lines do not appear. The stellar source was the B2 star α Virginis, α (1900) $13^h\ 19^m{\cdot}9$, δ − 10° 38′. The trigonometric parallax of this star is +0″·011 ± 0·010 and the spectroscopic parallax is 0″·024, indicating that

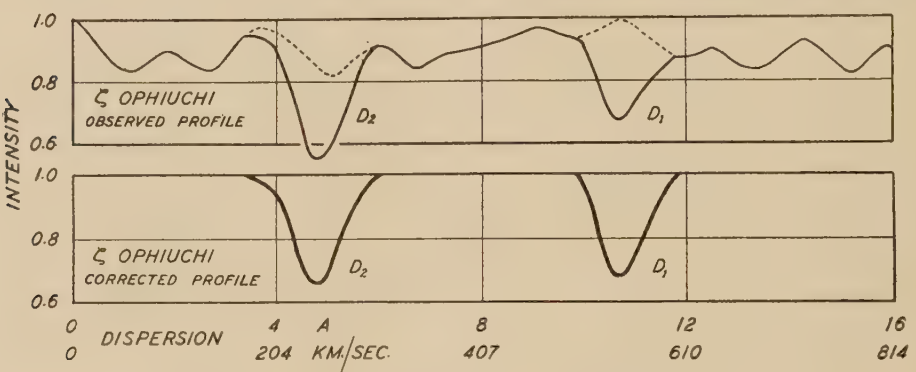

Fig. 1.—*Corrections for effect of water-vapour lines.*

it is probably within 100 parsecs.† While the absence of stellar or interstellar sodium lines of appreciable intensity in this spectrum seemed reasonably certain, the matter was checked by observing the water-vapour bands in the spectrum of a searchlight situated at the Esquimalt Naval Station about 10 km. distant. The searchlight was treated as an artificial star, and, with the aid of an auxiliary mirror, the spectrum was observed with the same spectrograph as was that of α Virginis. With the aid of spectra from these two sources it was possible to reconstruct the part of the band profile ordinarily obscured by the interstellar lines. Fig. 1 shows a profile of the D lines in the spectrum of ζ Ophiuchi with the accompanying water-vapour band and the corrected profile of the D lines below. The profiles are uncorrected for finite resolving power and are accordingly largely influenced by the properties of the instrument. The difficulties of adopting a more rigorous procedure seem insurmountable with the dispersion at present available, but since total absorptions are involved it seems improbable that the order of magnitude of the corrections would be greatly different in any case. As may be seen from the figure, one effect of the correction is to diminish the value of the intensity ratio D2/D1. Owing to the approximate nature of the process the writer does not feel any great confidence in the value of this ratio when the water-vapour absorption is of appreciable intensity relative to the interstellar lines. On the other hand, when comparisons are made between Ca^+ K and *Na* D2, it can be stated with reasonable

* *Zs. f. Phys.*, 81, 465, 1935.

† *Gen. Cat. Stellar Parallaxes*, Schlesinger and Jenkins, 1935.

confidence that the intensity of the sodium line is not over-estimated. To date corrections have been applied to only two stars, ι Orionis and ζ Ophiuchi. For other stars so far observed the effect is negligible.

3. *Relation between Intensity and Velocity.*—The new theoretical developments outlined in the introduction to this paper suggest a fairly definite correlation between intensity and velocity. For the purposes of this discussion we limit the term "velocity" to the residual radial velocity shown by the interstellar line after the solar motion has been removed. The term "intensity" refers to the equivalent width of the interstellar line in km./sec.

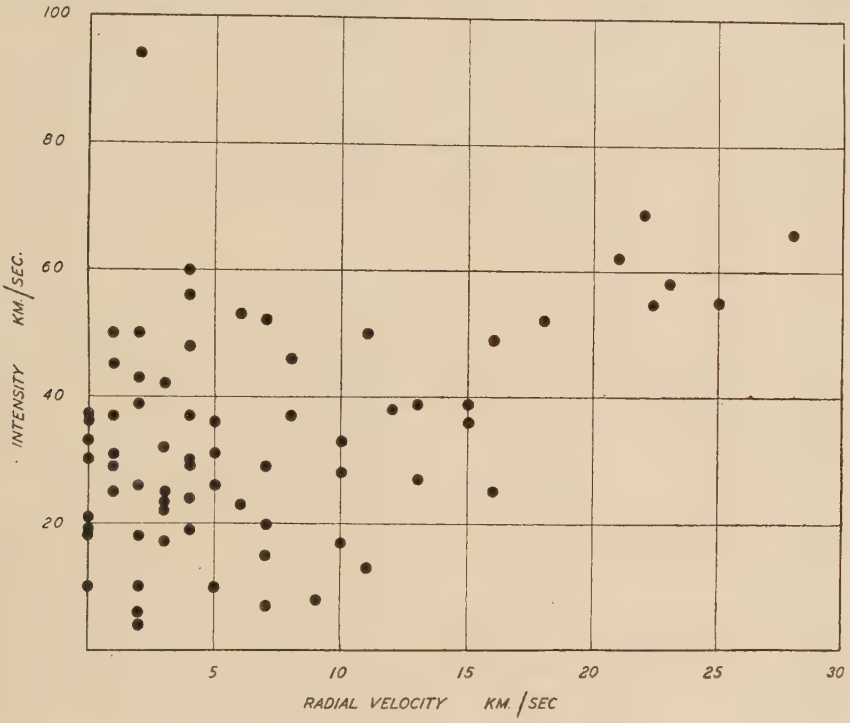

FIG. 2.—*Relation between intensity and radial velocity.*

As has already been mentioned, Eddington's calculations indicate that, in so far as the line profile is due to galactic rotation, the intensity should be approximately equal to twice the velocity. Accordingly the simultaneous broadening and displacement of the line due to galactic rotation should give rise to a combined distance and longitude effect which might reasonably be expected to show itself as a relation between intensity and velocity.

In fig. 2 are plotted all observations so far available of the intensities and velocities of interstellar lines, while Table IV contains the same data in tabular form. The material consists of measures of 69 stars. Intensities for 53 of these have been determined by the writer, while the remainder are due to Williams.* The velocities are due to Plaskett and Pearce.† The intensities appear as ordinates while the velocities are the abscissæ.

* *Ap. J.*, 79, 229, 1934. † *Pub. D.A.O.*, V, No. 3, 1933.

TABLE IV

Relation between Line Intensity and Residual Radial Velocity

Star (H.D.)	No. Plates	Int. km./sec.	Vel.	Star (H.D.)	No. Plates	Int. km./sec.	Vel.
108	7	38	−12	190864	3	56	+ 4
1337	4	36	−16	190918	5	50	+ 1
5005	1	50	−11	191201	1	36	+ 5
13841	5	55	−25	191765	5	39	+ 2
13854	5	55	−23	192103	3	43	+ 1
14134	5	66	−28	192163	5	36	− 1
14143	4	69	−23	192422	1	39	+13
14818	7	62	−21	192639	3	24	+ 4
14956	6	52	−18	192641	1	31	0
17505	1	49	−16	193237	8	33	+ 2
23180	3	8	+ 5	193322	1	37	+ 4
24912	6	15	+ 8	193514	1	45	+ 1
28446	2	19	− 6	193793	4	50	− 2
30614	5	29	− 4	194279	2	60	+ 4
34656	1	42	− 3	194839	1	53	− 6
36486	3	4	− 1	195592	1	32	+ 3
36861	5	17	− 3	198478	2	37	+ 8
37043	3	13	+12	198846	3	23	+ 6
37128	6	10	0	199579	1	26	+ 5
37468	1	6	− 2	202214	1	31	− 5
37742	3	7	− 7	203064	7	26	+ 1
47129	1	29	0	206165	1	33	+11
47839	7	19	+ 1	208392	1	37	− 1
91316	12	10	+ 1	208947	2	22	− 3
149757	3	6	+ 3	209481	1	31	+ 2
149881	5	25	+ 3	209975	4	21	− 1
166182	2	18	0	310809	1	46	− 8
168957	2	22	+ 3	210839	12	25	− 2
184915	1	17	+10	212455	1	94	− 2
186980	1	29	+ 4	212571	1	10	− 4
188001	3	27	+13	213420	1	18	+ 2
188209	4	28	+10	214680	1	20	− 6
190429 N	7	52	+ 7	217297	1	37	0
190429 S	5	48	+ 4	218915	4	39	−15
190603	1	30	+ 4	224151	1	25	−16

The plotted points of fig. 2 give some indication of the expected general correlation between intensity and velocity, but this relationship is by no means as definite and clear-cut as might be expected. In fact, if six stars in the Perseus region were omitted from the diagram the actual existence of any correlation would be rendered uncertain. It is probable that more extensive observational material, particularly for the fainter stars, would considerably improve the situation, but in any case the large spread in intensity suggests that in many cases some mechanism other than galactic rotation exercises an important influence in the formation of the line profile.

Any reasonable explanation of these observations would seem to require the existence of some irregularity either in the motion or the distribution of interstellar material. The hypothesis is accordingly advanced that the profiles of interstellar lines are in some cases largely a result of irregular motions of condensations or discontinuous parts of the interstellar cloud; and in the remaining sections of this paper additional evidence is presented in support of this view.

4. *Evidence for Peculiar Motion from the Interstellar Velocities of Cygnus and Perseus Clusters.*—In connection with the above-mentioned lack of any very marked correlation between intensity and velocity, it seems worth while to consider in some detail the observational data relating to the two well-known groups of stars, the double cluster h and χ in Perseus, α (1900) $2^h\ 12^m.0$, δ 56° 40′, and the cluster N.G.C. 6871 in Cygnus, α (1900) $20^h\ 2^m.1$, δ 35° 30′. Taking the six brightest stars of each group and using velocities and constants of galactic rotation due to Plaskett and Pearce, we can derive the distance of each cluster by the use of the well-known galactic rotation formula *

$$2\rho = \bar{r}A \sin 2(l - lo) \cos^2 b,$$

and derive values of the mean absolute magnitudes of the stars concerned. Table V gives the velocity and intensity data for individual stars, while the mean values and the results of the calculations are given in Table VI.

It will be seen from Table VI that the distance of the Perseus cluster comes out over three times that of the cluster in Cygnus, resulting in mean absolute magnitudes for the stars of -5.4 and -2.4 respectively. The difference of 3·0 magnitudes between the two sets of values seems unduly large, in spite of the tendency among early type stars to show a large dispersion in absolute magnitude, and there accordingly appears reason to believe that this method has led to spurious estimates of distance. This conclusion receives independent support from the work of Trumpler,† whose estimates of distance are based both on the apparent diameters of the clusters and on the apparent magnitudes and spectral types of the stars. After applying corrections for space absorption he derived a distance of 1330 parsecs for the Perseus stars and 1340 parsecs for the cluster in Cygnus.

The results of Table VI were derived on the assumption that the velocities of interstellar lines are determined solely by galactic rotation, acting according to the simple formula $2\rho = \bar{r}A \sin (l - lo) \cos^2 b$. The anomalous values of distance found in Table VI suggest that this assumption may not be entirely valid and that there may exist peculiar motions of considerable magnitude associated perhaps with some irregularity of distribution of interstellar material. Such peculiar motions, especially if they occurred on a fairly large scale, might easily lead to errors in calculations of distance and in the same way they would tend to obscure the relation between velocity and intensity. It is of interest to note that Struve ‡ has been led to somewhat similar conclusions from a study of the velocities and distances of the stars 9 Cam and 19 Cep.

* *Pub. D.A.O.*, **5**, 197, 1933. † *L.O.B.*, **14**, 154, 1930. ‡ *Ap. J.*, **79**, 275, 1934.

TABLE V

Data for Individual Stars from Perseus and Cygnus Clusters

Perseus Stars	R.A. (1900)	Dec. (1900)	Type	Mag.	Interstellar Velocity	Intensity
	h m	° ′				
H.D. 13841	2 9·8	56 34	B2	7·2	−25	55
,, 13854	2 9·8	56 36	Bo	6·4	−23	58
,, 14134	2 12·1	56 40	B2	6·7	−28	66
,, 14143	2 12·2	56 43	B1	6·7	−23	69
,, 14818	2 18·2	56 10	B1	6·2	−21	62
,, 14956	2 19·6	57 14	B1	7·3	−18	52
		Mean Values	B1	6·75	−23	60
Cygnus Stars						
H.D. 190429*	19 59·8	35 45	O7	7·3	+5	50
35° 3955	20 2·2	35 33	B1	7·8	+4	
H.D. 190918	20 2·2	35 31	O9	7·0	+1	50
,, 190919	20 2·2	35 24	B1	7·3	+5	
,, 191201	20 3·6	35 26	Bo	7·1	+5	36
,, 191561	20 5·5	35 11	B2	7·7	+1	
		Mean Values	Bo	7·37	+3·5	45

* Visual double. Average values of spectral type, magnitude and velocity given.

TABLE VI

Distances, Perseus and Cygnus Clusters

	Perseus h and χ	Cygnus N.G.C. 6871
Long. Gal. Centre (lo)	325°	325°
Constant A (km./sec./parsec)	0·017	0·017
Gal. Long. l	103°	41°
Gal. Lat. b	3°	0
Velocity ρ	−23 km./sec.	+3·5 km./sec.
Distance $2\rho/A \sin 2(l-lo) \cos^2 b$	2720 parsecs	880 parsecs
Mean Apparent Mag.	6·75	7·37
,, Absolute Mag.	−5·42	−2·35
,, Spectral Type	B1	Bo
,, Interstellar Intensity	60 km./sec.	45 km./sec.
Distance (Trumpler)	1330 parsecs	1340 parsecs

On the basis of these general ideas the average intensities of the interstellar lines in the spectra of the Perseus and Cygnus stars are not inconsistent with approximately equal distances for the two clusters. The differences in intensity could be accounted for partly as a true longitude effect

of the type predicted by Eddington and partly as a result of obscure irregularities of motion and distribution in the interstellar gas.

5. *The Doublet Intensity Ratios K/H and D2/D1.*—Previous measures by the writer * indicated that the intensity ratio D2/D1 for the interstellar lines of sodium reached a value of 2·00 for one star, ρ Leonis, with smaller values for stars with stronger lines. In the same paper a few values of the ratio K/H for calcium were included, but owing to the low dispersion employed and the small number of observations these measures were given low weight. The ratio K/H has been determined for 61 stars by Williams.† From total absorptions he derives an average value of K/H equal to 1·83. From depths alone this value becomes 1·61. When only stars with diffuse hydrogen lines are considered a further reduction to 1·47 is obtained. It should be noted that this is an average value, and that values for individual stars may be considerably higher or lower. Williams‡ has also determined the ratio K/H for Nova Herculis under conditions which appeared favourable for accurate measurement. The value derived was approximately 2·00.

Owing to the theoretical importance of the doublet ratio the writer has devoted a considerable amount of time to the observation of a number of bright stars, using the highest dispersion available, 4·5 and 6·5 A. per mm. With this dispersion the wing of the hydrogen line can be drawn in with relatively little ambiguity, especially since the interstellar lines of the objects chosen are relatively weak, while the hydrogen lines are broad and diffuse. Some degree of error, owing to the blend with $H\epsilon$, is no doubt still involved, but there appears reason to believe that the general order of magnitude of the ratio K/H will not be greatly in error. In this connection it may be noted that the value of the ratio for Nova Herculis, where there is a high probability that the measure is not complicated by blends, is in good agreement with that for other stars where the absolute intensities of the lines are of the same order. In Table VII are collected measures of the K/H ratio for 15 stars. With the exception of four W stars, indicated by asterisks, all the measures have been made with 3-prism dispersion 4·5 or 6·5 A. per mm. Probable errors are included in this table and in Tables VIII and XI, where three or more observations are available.

Measures of the ratio D2/D1 for sodium will be found in Table VIII. With the exception of two stars (ζ Oph and ι Ori) of low declination in which the water-vapour lines are strong the measurement of this ratio has occasioned no special difficulty. The close proximity of the two lines should make for accuracy in comparisons of intensity, and it is probable that for lines of reasonably large intensity (10 km./sec. or greater) the value of the doublet intensity ratio is as reliable as that for Ca^+.

Summarizing previous discussions of the interpretation of the doublet intensity ratio,§ it may be pointed out that, owing to the low density of interstellar material, the mechanism of line formation is probably restricted

* *M.N.*, **94**, 663, 1934. † *Ap. J.*, **79**, 280, 1934. ‡ *M.N.*, **95**, 573, 1935.
§ Schultz, *Zs. f. Astrophysik*, **1**, 300, 1930; Unsöld, Struve and Elvey, *Zs. f. Astrophysik*, **1**, 314, 1930.

TABLE VII

Doublet Intensity Ratio, Ca+

Star	H.D.	Type	No. Plates	Int. 3933	Int. 3968	Ratio K/H
ξ Per	24912	O7	4	9·0 ± 0·3	5·5 ± 0·2	1·64
δ Ori	36486	Bo	5	3·6 ± 0·3	1·8 ± 0·2	2·0
ι Ori	37043	O8	4	6·2 ± 0·2	3·5 ± 0·2	1·8
ϵ Ori	37128	Bo	4	9·1 ± 0·3	4·4 ± 0·2	2·1
ζ Ori	37742	Bo	3	7·0 ± 0·4	3·7 ± 0·4	1·9
S Mon	47839	O7	2	14·0	8·0	1·7
ρ Leo	91316	Bo	10	8·4 ± 0·2	4·6 ± 0·2	1·8
ζ Oph	149757	Bo	1	5·0	2·4	2·1
N Her			8	7·0 ± 0·2	4·1 ± 0·2	1·8
κ Aql	184915	Bo	1	17·0	7·0	2·5
35° 4001*	191765	W6	5	37·8 ± 1·6	26·9 ± 3·0	1·4
35° 4013*	192103	W'7	3	43·0 ± 5·0	30·3 ± 2·0	1·4
37° 3821*	192163	W6	5	35·6 ± 2·2	32·4 ± 1·0	1·1
36° 3956*	192641	W'7	1	31·0	27·0	1·15
λ Cep	210839	O6	1	21·4	18·1	1·1

* Measures made with single prism dispersion.

TABLE VIII

Doublet Intensity Ratio, Na

Star	H.D.	Type	No. Plates	Int. 5889	Int. 5895	Ratio D2/D1
62° 2363	108	O6fq	2	50·5	39·5	1·3
Boss 46	1337	O8	1	50·9	42·0	1·2
56° 530	14143	B1	1	77	66	1·2
10 Per	14818	B1	3	57·5 ± 1·6	43·6 ± 2·2	1·3
ξ Per	24912	O7	5	17·4 ± 1·0	14·7 ± 0·5	1·2
9 Cam	30614	O9	7	32·0 ± 2·2	27·4 ± 1·6	1·2
37° 1146	34656	O6	1	42·0	31·0	1·35
λ' Ori	36861	O8	6	17·6 ± 0·7	13·4 ± 0·5	1·3
ι Ori	37043	O8	3	9·5 ± 1·0	7·0 ± 1·5	1·4
ϵ Ori	37128	Bo	3	21·0 ± 2·0	12·0 ± 2·0	1·75
ρ Leo	91316	Bo	6	10·7 ± 0·5	5·3 ± 0·6	2·0
ζ Oph	149757	Bo	5	12·0 ± 1·4	8·0 ± 2·0	1·5
102 Her	166182	B2	1	12·0	6·0	2·0
κ Aql	184915	Bo	1	21·0	18·0	1·2
Boss 5082	188209	O8	2	33·0	30·1	1·1
35° 3930	190429N	O5	2	69·5	72·5	0·96
35° 3953	190918	O9W5	1	55·0	44·0	1·25
36° 3958	192639	O7f	1	52·9	45·0	1·2
P Cyg	193237	B1q	6	24·5 ± 1·1	26·6 ± 0·6	0·92
43° 3571	193793	O5W8	4	60·5 ± 0·8	51·2 ± 1·8	1·2
A Cyg	203064	O8	7	27·5 ± 2·4	21·7 ± 2·1	1·3
14 Cep	209481	O9	3	38·4 ± 1·3	33·3 ± 0·6	1·15
19 Cep	209975	O9	4	36·0 ± 1·6	30·2	1·2
λ Cep	210839	O6e	9	41·7 ± 1·6	36·0 ± 1·4	1·15

to so-called classical radiation damping combined with some form of Doppler broadening. Assuming an appreciable initial Doppler width to the line the maximum value for the doublet ratio (2 : 1) is reached only when the number of absorbing atoms in the line of sight is relatively small. As the thickness of the absorbing layer is increased either by greater density or increasing distance, the lines tend to become saturated and the intensity ratio approaches the value unity. If the optical depth be then further increased to the point where the line width due to radiation damping wings becomes great compared to the Doppler width, the ratio will again increase and approach asymptotically the value $\sqrt{2}$ corresponding to a mechanism of pure radiation damping.

In a previous paper the writer * presented evidence suggesting that the above entire cycle of values of the doublet ratio was traversed by the sodium D lines. Subsequent observations have not entirely confirmed this view. The data of Tables VII and VIII indicate that the maximum value of 2·0 is reached and approached for the nearer stars, and that, for some of the more distant objects with stronger lines, the ratio falls to a value not far from 1·0. Whether or not there is actually a minimum value of 1·0 with a subsequent increase to $\sqrt{2}$ remains uncertain at present, and will probably be decided only when considerably more extensive observational data are available.

The most important result of the present investigation of the doublet ratio is undoubtedly the high maximum value (of the order of 2·0) obtained for the nearer stars. In so far as reliance may be placed on the observations this result indicates that absorption has not reached the point of saturation, and that consequently the density of interstellar material is considerably lower than has hitherto been supposed. Moreover, when considered in connection with the absolute intensities of the lines, the high maximum value of the doublet ratio may provide a valuable clue to the mechanism of line formation. It is required to ascertain the circumstances under which an unsaturated line can give rise to the intensities which are observed.

In the absence of evidence for saturation we may assume that to a first approximation the mechanism is restricted to some form of Doppler broadening. The following three possibilities may be considered : (1) differential velocity due to galactic rotation ; (2) temperature motions of the atoms ; and (3) irregular motions of groups of atoms or of condensations or discontinuous parts of the interstellar cloud.

If the case of the four Orion stars of Table VII be considered we may note that their distance is of the order of 100 parsecs, so that (1) could not produce a line of intensity of more than 2 km./sec. even under conditions of complete saturation. Temperature motions of the atoms may be somewhat more effective. As Eddington has already pointed out, at 15,000° the radial velocities of the atoms of Ca^+ follow the distribution law $e^{-h^2v^2}$ where $h^2 = 0{\cdot}16$. Assuming that the form of the line is determined by temperature motions alone, it is possible to make a rough calculation of the doublet ratio corresponding to different values of the line intensity.

* *M.N.*, **94**, 663, 1934.

Let $\tau = \tau_0$ be the value of the optical depth in the centre of the line. Then at any point in the line corresponding to velocity v the value of τ will be

$$\tau = \tau_0 e^{-0.16v^2}, \tag{1}$$

and the intensity across the line will be represented by the formula

$$I_v = e^{-\tau_0 e^{-0.16v^2}}, \tag{2}$$

where I_v represents the intensity of light at any point in the line relative to the continuous spectrum.

The total absorption of the line may be derived by plotting the function $r = (1 - e^{-\tau})$, where τ has the value given in (1), and measuring with a planimeter the area enclosed between the curve and the axis of abscissæ. This has been done for values of τ_0 ranging from 1·2 to 0·15, and the results will be found in Table IX. Values of τ_0 are given in the first column of this table, while in the second column are included the corresponding values of the total absorption (equivalent width in km./sec.). In the third column are values of the doublet ratio K/H. To compute this ratio we take τ_0 for the K line and $\tau_0/2$ as the value for the H line corresponding to the relative "oscillator strengths" for the two lines.

TABLE IX

Computed Values, Doublet Intensity Ratio Ca^+

τ_0	I (km./sec.)	Doublet Ratio K/H
1·2	3·57	1·66
1·0	3·15	1·70
0·8	2·69	1·745
0·7	2·43	1·76
0·6	2·15	1·79
0·5	1·85	1·83
0·4	1·54	1·88
0·35	1·37	1·95
0·30	1·20	2·00
0·25	1·01	...
0·20	0·82	...
0·15	0·60	...

While these results are necessarily approximate, the degree of error involved can scarcely be very large, and a comparison of columns 2 and 3 should give a fairly accurate idea of the run of intensities corresponding to particular values of the doublet ratio. It will be seen that for values of the doublet ratio in excess of 1·8 the equivalent width of the line does not exceed 2·0 km./sec. Comparing these figures with the observed values we find that, while the average value of the doublet ratio for the brighter stars of Table VII is between 1·8 and 2·0, the absolute intensities are of the

order of 6 to 7 km./sec., over three times the computed value. From this it may be concluded that the observed intensities are too great to be produced by temperature motions alone. There remains the effect of irregular motions of groups of atoms or condensations within the interstellar cloud, and such a mechanism appears to the writer to offer a very attractive possibility for the solution of the difficulty. We require a Doppler width to the lines sufficient to account for intensities of 10 km./sec. or greater, without approaching saturation in the line centres. This requirement can, of course, be fulfilled by irregular motions of parts of the cloud, provided their magnitude and distribution are at our disposal. As has already been pointed out, an analysis of the velocities and intensities of interstellar lines suggests the existence of irregular or peculiar motions in the cloud. Even in the absence of such independent evidence, however, the data of Tables VII and IX would appear to find their most logical explanation in the existence of irregular motions of the necessary magnitude.

Of some interest in this connection are the observations by Buisson, Fabry and Bourget,* and by Campbell and Moore † of irregular motions in the Orion Nebula. They have observed differences of velocity of as much as 13 km./sec. for different parts of the nebula. Moreover, Campbell and Moore have drawn attention to the existence of lines broadened and deformed as though the velocity varied in an irregular manner throughout the depth of gas contributing to the intensity of the line. There is, of course, no certain correlation between nebulosity such as that of the Great Nebula in Orion and matter producing the interstellar lines, but the observation of irregular motion in the nebula is at least suggestive of what might take place elsewhere in interstellar space, while the general order of magnitude of the observed velocities is not far from the required amount.

The discussion of the doublet ratio has so far referred mainly to the calcium lines, although the same theoretical considerations apply in a general way to both sodium and calcium. Temperature motions will have a somewhat greater influence on the sodium lines, but it is doubtful whether the difference would be great enough to affect appreciably the observations. Difference in optical thickness is probably the main factor in determining comparative values of the doublet ratio. Unfortunately observations relating to this point are less adequate than might be desired. There are, however, six stars in which the ratio has been observed in common, average values being 1·9 for Ca^+ and 1·5 for *Na*. Since a lower value of the doublet ratio may be assumed to indicate a closer approach to saturation, the conclusion follows that the optical depth for sodium may be actually greater than for Ca^+. In view of the meagre observational material this result is naturally open to question, but we shall see later that a direct comparison of sodium and calcium intensities leads to a similar result.

6. *Complex Lines which may be Interpreted as due to Interstellar Absorption.*—If, as has been suggested, the profiles of interstellar lines are, in some cases, largely a result of irregular motions within the cloud involving a

* *Comptes Rendus*, **158**, 1269, 1914; *Ap. J.*, **40**, 241, 1914.
† *Pub. Lick Obs.*, **13**, 96, 1918.

certain amount of discontinuity between one part of the cloud and another, then it would appear reasonable to expect irregular structure or asymmetry sometimes to appear in the observed profiles. Irregularity of structure which may be interpreted in this way has actually been observed in the narrow H and K lines of the stars λ Leonis, ε Orionis and ζ Orionis, while asymmetry is suspected in the stars δ Orionis and ι Orionis. In fig. 3 are shown the profiles of K observed in the spectra of the first three stars with the three-prism spectrograph (dispersion 4·5 and 6·5 A. per mm.). In each case a

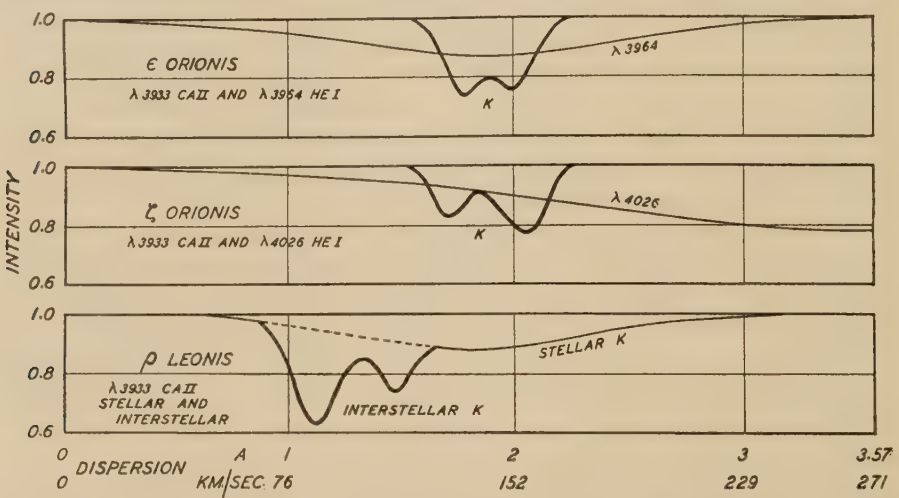

Fig. 3.—*Narrow complex K lines (believed to be interstellar) compared with stellar lines. The HeI line λ 4026 in the spectrum of ζ Orionis is so wide that only half the profile could be included. (The HeI lines are not in their normal positions relative to K.)*

stellar line is shown for purposes of comparison. The profile of K in the spectrum of ρ Leonis consists of a broad dish-shaped line, presumably stellar, on the violet wing of which are superposed two narrow lines, partly blended, which in their widths, shapes and velocities show no obvious relationship to other lines in the spectrum of the star. In the first observations of these lines the fainter one was regarded as a flaw in the photographic emulsion, but further investigation showed beyond doubt that it was real. The velocities shown by the two narrow lines, presumed to be interstellar, are −11 and +16 km./sec. respectively, while the stellar velocity is +42 km./sec. The stellar lines in other parts of the spectrum in width and form are similar to the stellar K of fig. 3. The dispersion available for examining the sodium D lines is not sufficient to determine definitely whether or not they exhibit similar structure, but their mean velocity, −1·4 km./sec., agrees well with the weighted mean of the H and K lines.

For the two stars ε Orionis and ζ Orionis the principal reason for attributing the complex lines to interstellar absorption is their narrowness. The stellar lines of these two stars are unusually broad and diffuse, suggesting rapid rotation, so that difficulties are encountered in any attempt to account for the narrow lines as arising in the stellar atmosphere. For these two stars evidence afforded by the relative velocities of stellar and (presumed)

TABLE X

*Radial Velocities * from Minima of Complex Lines*

Star	Ca^+ Vel.	Weight	*Na* Vel.	Stellar Vel.
ϵ Orionis	v + 8·2	1	...	...
	r +22·8	1	...	...
Mean	+15·5		+18·2	+25·8
ζ Orionis	v − 3·0	1	...	...
	r +23·0	2	...	...
Mean	+14·0		+17·0	+18·8
ρ Leonis	v −11·0	6·3	...	...
	r +17·0	2·5	...	...
Mean	− 1·4		− 1·4	+42·2

* These velocities include the solar motion.

interstellar lines is uncertain, as both are close to the reflex of the solar motion. Velocity data for the three stars ρ Leonis, ϵ Orionis and ζ Orionis are given in Table X. This includes the velocities of the stellar lines and velocities obtained by setting on the minima of the complex "interstellar" lines. It will be noted that the velocity data for ϵ Orionis and ζ Orionis are consistent with a stellar origin for the red component of the narrow lines, although, for reasons given above, the writer prefers the alternative explanation of interstellar absorption.

7. *The Relative Intensities of Calcium and Sodium Lines.*—According to the theory discussed in the introduction to this paper the sodium lines may be approximately equal to, but not greater than, those of calcium. Providing the density is sufficiently great to result in saturation over the flat part of the line, differences in intensity will be due to the effect of wings, whatever may be their origin. If theoretical values for the relative numbers of Ca^+ and *Na* atoms be accepted, then the 300-fold greater abundance of Ca^+ should result in stronger wings on the lines. Accordingly, the calcium lines would be expected always to be somewhat stronger than those of sodium.

Reasons have already been given for believing that the above conditions may not be universally present in interstellar space. In particular, evidence has been presented suggesting, not only that the density of Ca^+ may be less than had formerly been supposed, but that the optical thickness of *Na* for the D lines may be actually greater than that of Ca^+ for K absorption. In Table XI further evidence, pointing in the same direction, is provided by a direct comparison of the intensities of sodium and calcium lines. This table is based on reasonably extensive observational material, over 200 plates

TABLE XI

Relative Intensities Ca^+ and Na

Star	H.D.	Type	Int. Ca^+ (K)	No. Plates	*Na* (D2)	No. Plates	Ratio D2/K
62° 2363	108	O6fq	37·6 ± 1·8	7	50·5	2	1·35
Boss 46	1337	O8	36·0 ± 1·9	4	50·9	1	1·41
56° 530	14143	B1	69·4 ± 3·0	4	77·0	1	1·12
10 Per	14818	B1	61·1 ± 1·1	6	57·5 ± 1·6	3	0·94
ξ Per	24912	O7	14·6 ± 0·6	17	17·4 ± 1·0	5	1·19
9 Cam	30614	O9	28·5 ± 1·6	5	32·0 ± 1·6	7	1·12
λ Ori	36861	O8	16·7 ± 1·0	5	17·6 ± 0·7	6	1·05
ι Ori	37043	O8	*6·2 ± 0·2	4	9·5 ± 1·4	3	1·53
ε Ori	37128	Bo	*9·1 ± 0·3	4	21·0 ± 2·0	3	2·30
ρ Leo	91316	Bo	*8·4 ± 0·2	10	10·7 ± 0·5	6	1·27
ζ Oph	149757	Bo	*5·8 ± 0·6	3	12·0 ± 2·0	3	2·07
102 Her	166182	B2	18·0	2	12·0	1	0·67
κ Aql	184915	Bo	17·4	1	21·0	1	1·20
Boss 5082	188209	O8	27·7 ± 1·3	4	33·0	2	1·19
35° 3930	190429N	O5	52·4 ± 0·8	7	69·5	2	1·33
35° 3953	190918	O9 W5	49·6 ± 1·8	5	55·0	1	1·11
36° 3958	192639	O7f	24·0 ± 1·0	3	52·9	1	2·20
P Cyg	193237	B1q	34·4 ± 0·6	8	24·5 ± 1·1	6	0·71
43° 3571	193793	O5 W8	50·0 ± 1·1	4	60·5 ± 0·8	4	1·21
A Cyg	203064	O8	25·4 ± 1·9	7	27·5 ± 2·4	7	1·08
19 Cep	209481	O9	20·6 ± 1·4	4	36·0 ± 1·6	3	1·75
λ Cep	210839	O6e	26·3 ± 1·1	12	41·7 ± 1·6	9	1·58

* Measures made with high dispersion 4·5 and 6·5 A. per mm.

in all. Precautions for avoiding systematic errors have already been described. The results indicate an average value of 1·39 for the ratio D2/K, so the sodium lines on the basis of this determination are very considerably stronger than those of calcium. This result is in general agreement with previous determinations by the writer, and the tendency shown by recent observations is to increase rather than to decrease the intensity of the sodium lines relative to those of calcium. Further, there seems to be a fairly definite tendency for the ratio D2/K to be greater for stars with faint lines. This is shown particularly for the stars ε Orionis and ζ Ophiuchi, where the intensities are unusually well determined, the Ca^+ lines being measured by both Williams and the writer (Tables II and III). Since, in the absence of saturation, differences in intensity tend to be greater for weaker lines, the observation of a higher value of the ratio D2/K for lines of small intensity forms a satisfactory complement to other observational evidence, suggesting that the absorbing sodium atoms actually outnumber those of Ca^+.

In conclusion I wish to thank Mrs. Miriam Beals, who has done a large part of the work of measurement and reduction of tracings, and Dr. A.

McKellar, who prepared the drawings which accompany this paper, as well as giving valuable assistance in the observation of the water-vapour spectrum. I am also very greatly indebted to the officer in charge and staff of the torpedo department, Esquimalt Naval Station, whose kind co-operation in the mounting of a searchlight made possible observations of the water-vapour bands under the difficult weather conditions of the winter season.

Dominion Astrophysical Observatory,
Victoria, B.C.:
1936 *March* 10.

THE VARIABLE SPECTRUM OF γ CASSIOPEIÆ.

(Fifth Paper.)

William J. S. Lockyer, M.A., Ph.D., Major (late R.A.F.).

In continuation of my previous papers * on the variable spectrum of γ Cassiopeiæ (H.D. 5394, R.A. $0^h\ 50^m.7$, Dec. $+60°\ 11'$, Ptg. Mag. 2·01, Sp. Type Boemn7) further photographs have been taken with the same instrument as before, covering the period of time between 1935 October 29 and 1936 April 22. Further new developments in the spectrum have taken place, which relate to the change of character and strength of some of the series of neutral helium and the central absorption of the hydrogen lines.

After the appearance of the strong neutral helium absorption line at λ 3888 on 1935 October 7 all the other neutral helium lines still maintained their diffuseness. It was not until a photograph was obtained about two months later, namely, on December 20, that some *sharp* and moderately strong helium lines appeared, corresponding to some of those which were previously diffuse (see Plate 8). These sharp lines belonged to two series of helium, namely, the λ 4471 and the λ 5015 series. The other two helium series, which still maintained their diffuseness, were those of λ 4713 and λ 4921.

The spectrum of γ Cassiopeiæ thus differentiates between two *groups* of series of helium, but in a manner which seems difficult to explain.

The series represented in γ Cassiopeiæ correspond to the multiplet designations of triplets $(2^3S - Z^3P°)$, $(2^3P° - Z^3D)$ and $(2^3P° - Z^3S)$ and of singlets $(2^1S - Z^1P°)$ and $(2^1P° - Z^1D)$.

One would have expected that *each group* of these series would behave similarly in γ Cassiopeiæ, but this is not the case. In the first group the series λ 3888 and λ 4471 have become sharp, while the series λ 4713 has remained diffuse. In the second group the series λ 5015 has also become sharp, while the series λ 4921 has remained diffuse. One would have thought that the series λ 4713 would have become sharp and the series λ 5015 remained diffuse.

* *M.N.*, **93**, 362, 619, 1933; **95**, 520, 1935; **96**, 2, 1935.

The following table sums up the nomenclatures of the five series of helium and their character in γ Cassiopeiæ :—

Series λ	Multiplets	Character in γ Cas.
3888·64	$2^3S - Z^3P^\circ$	Sharp
4471·47	$2^3P^\circ - Z^3D$	Sharp
4713·14	$2^3P^\circ - Z^3S$	Diffuse
5015·67	$2^1S - Z^1P^\circ$	Sharp
4921·92	$2^1P^\circ - Z^1D$	Diffuse

Another variation which occurred rather abruptly in the spectrum of γ Cassiopeiæ was first recorded on the Sidmouth plates on 1935 December 20, soon after the sudden appearance of the strong helium absorption line at λ 3888 on October 7. This change related to the great increase in the intensity of the central absorption of the hydrogen lines (see Plate 8), and corresponded in date to that of the sudden appearance of the sharp helium lines. So strong was this absorption that the bright hydrogen components stood out in great relief and could be seen extending far down into the violet to $H\tau$; the components of $H\beta$ were also clearly separated, which had not been the normal case previously.

The behaviour of these helium and hydrogen lines in γ Cassiopeiæ has its counterpart to some extent in another Be-type star, namely, ϕ Persei.

The Helium Line at λ 3888.—To take the case of the strong absorption line of helium at λ 3888 first. It was probably due to the appearance of this line in ϕ Persei that the hydrogen line $H\zeta$ (λ 3889·05) owed its peculiar behaviour, although its presence was not even then suspected (see Plate 8). In the discussion of the spectrum of this star special attention was drawn to the peculiar behaviour of $H\zeta$ in the following words :— *

"A most noteworthy feature that can be observed by examining different spectra of ϕ Persei as a whole is that, while $H\zeta$ does not seem to change in intensity at all in any of the negatives covering all phases, some spectra exhibit the double bright line $H\beta$ and the central absorption of $H\zeta$ as the most prominent and outstanding lines, while others display the central absorptions of $H\gamma$, $H\delta$, $H\epsilon$ and $H\theta$ so conspicuously that $H\zeta$ is no longer specially noticeable."

To satisfy myself on this point, direct comparisons have since been made with the recent γ Cassiopeiæ negatives and those of ϕ Persei taken with the same instrument in 1925–26. This shows undoubtedly that the helium line at λ 3888 was present and was responsible for the abnormality of the intensity of $H\zeta$ (see Plate 8).

The Character of the Helium Lines.—The case of some of the helium lines changing their characteristics, such as from "diffuse" to "sharp," occurred also in the spectrum of ϕ Persei. In the latter case I pointed out † that such a change was found to be cyclical with the period

* *M.N.*, 85, 598, 1925. † *M.N.*, 86, 492, 1926.

(126·8 days) of that star, the helium lines there examined changing from "diffuse" to "sharp" and *vice versa* at stated phases.

Here again the ϕ Persei negatives have been re-examined, since only the three helium lines at λ 4471, λ 4026 and λ 3964 were previously discussed. In this star the two series of helium, namely, λ 4471 and λ 5015, behave at certain phases of this star just as they do in γ Cassiopeiæ, changing from diffuse to sharp (and sometimes *vice versa* in the case of ϕ Persei).

At one phase of the star, namely, that at which the central hydrogen absorption is at a maximum, the series λ 5015, which includes the line at λ 3964, becomes diffuse, while λ 4471 maintains its sharpness (see Plate 8). On the other hand, the two helium series at λ 4713 and λ 4921 are always diffuse, as they have been in γ Cassiopeiæ up to the present time (1936 April 22).

Central Hydrogen Absorption.—Coming now to the third recorded change, namely, the great increase in intensity of the central absorption lines of hydrogen in γ Cassiopeiæ, this was very distinctly recorded by me in the spectrum of ϕ Persei in the following words *:—

"At some phases of this star the central absorption lines of the hydrogen series . . . increase in intensity and become very conspicuous, and at these phases the Balmer series of lines can be easily traced in the Sidmouth photographs down even to $H\rho$ (λ 3686·8), or even a little further."

There is no doubt that it is the central absorption of hydrogen that has increased in intensity and not the intensities of the bright components. If the latter had only occurred, then the tendency would have been for the sides of the bright lines nearer to the central absorption to have spread (photographically or otherwise) towards the central absorption and so diminished its intensity. The fact that the bright components of $H\beta$ are *clearly separated* in the later photographs, and not nearly blended as formerly was the case, points clearly to the intensification of the central absorption. It may be that *both* the central absorption and the bright components have become stronger, but if this be so then the central absorption must have increased at a greater rate than the bright components to give the actual appearance in the spectrum. The relative difference, however, remains the same.

A further recent examination of these ϕ Persei negatives brings out the fact that, as in γ Cassiopeiæ, the change from diffuseness to sharpness in the helium series is associated with the appearance of the strong central absorption lines of hydrogen, this association occurring in ϕ Persei at one particular phase of the stars period.

The Bright Hydrogen Components.—A few words may now be said as regards the recent behaviour of the bright components of the hydrogen lines. At the time of the appearance of the strong helium line at λ 3888 on 1935 October 7 the bright red components were much brighter than the violet components ($R > > V$). Since that date the violet components have been gradually brightening up, while the red components have been equally losing their intensity.

* *M.N.*, **86**, 486, 1926.

It is interesting to note that these sequences of changes of relative intensity of the bright hydrogen components seem to take place earlier in the ultra-violet than they do towards the red end of the spectrum. Thus, for example, a spectrum taken on 1936 April 9: while, in the cases of $H\beta$ and $H\gamma$, the red components were *much greater* than the violet components ($R >> V$); in the case of $H\delta$ the red component was only a *little greater* than the violet component ($R > V(L)$); further, in the case of $H\epsilon$, the *violet* component was *greater* than the red component ($V > R$). This procedure is, however, not restricted to γ Cassiopeiæ, because it had been noticed by me in other Be-type stars. Thus it was distinctly observed and recorded in the case of the spectrum of φ Persei. In my first paper on this star * I wrote: "It will be noticed that the differences in intensity between the components of $H\beta$ were always greater than those of $H\gamma$, and the same may be said about $H\gamma$ in relation to $H\delta$, although no actual measures were made. This diminishing difference of intensity between the components of the hydrogen lines seems to be a general feature of all the bright-hydrogen-line stars that have been examined."

In the case of γ Cassiopeiæ these differences seem to be very much more accentuated, so much so that the brighter of the two components ($R > V$) at the longer wave-lengths becomes the lesser bright component ($V > R$) at the shorter wave-lengths of the spectrum, and this in such a short distance along the spectrum, namely, from $H\delta$ to $H\epsilon$.

It seems difficult to account for these progressive different relative intensities of the components of the bright hydrogen lines from the short to the longer wave-lengths. It is a common feature of the hydrogen lines to become gradually less intense as the violet is approached, and it would normally be expected that, while the bright components of each of the hydrogen lines as a whole would decrease in intensity, as they undoubtedly do, the *relative* intensity of these bright components would remain the same. That they do not remain the same, but behave in the manner described above, needs explanation.

Similarity between γ Cassiopeiæ and φ Persei.—There appear therefore to be great similarities between the behaviour of the helium and hydrogen lines in the spectra of γ Cassiopeiæ and φ Persei. In the case of the latter the lines change cyclically, but this is not so with γ Cassiopeiæ, so far as the records go at present, because the *great changes* which have recently been taking place in the spectrum, both as regards the bright components of the hydrogen lines and the helium absorption lines, only date back to about the year 1932.

Up to the time of writing (1936 April 22) all the *sharp* helium lines and also the central absorption of hydrogen have considerably increased in intensity and have now become very prominent features of the spectrum.

It is too early yet to seek a cause for this peculiar inter-behaviour of the helium series and that of the central absorption of hydrogen since it is possible that other changes may occur in the near future which might not be satisfied by it. There are several broadening mechanisms available, such as abun-

* *M.N.*, 85, 583, 1925.

(1) γ Orionis. (2) γ Cas. 10 Sept. 1935. (3) 29 Oct. 1935. (4) 15 Mar. 1936. (5) ϕ Persei.

W. J. S. Lockyer, The Variable Spectrum of γ Cassiopeiæ.

dance, Doppler effect including axial rotation and pulsation, and the Stark effect, and these have been very closely studied by Otto Struve in relation to the spectra of the B- and A-type stars and his results published in several communications.* Which of these mechanisms, or modifications of them, can be applied to the phenomena represented in the spectrum of γ Cassiopeiæ remains for the future to decide.

EXPLANATION OF PLATE 8

This plate illustrates three distinct changes which have recently occurred in the spectrum of γ Cassiopeiæ.

No. 1. Comparison spectrum of γ Orionis showing different series of neutral helium.

No. 2. γ Cassiopeiæ (1935 September 10), showing all helium lines diffuse and absence of λ 3888.

No. 3. γ Cassiopeiæ (1935 October 29), showing helium line at λ 3888 very strong and sharp; all other helium lines diffuse.

No. 4. γ Cassiopeiæ (1936 March 15), showing three series of helium very sharp, and very strong central absorption of hydrogen.

No. 5. ϕ Persei (1925 September 20), illustrating a similar condition to the spectrum of No. 4 at one of the phases of this star, demonstrating also the great strength of λ 3888.

The five series of helium are indicated below the spectra. Those marked S (sharp) are the series which have recently become sharp, while those marked D (diffuse) have remained diffuse, *i.e.* suffered no alteration.

All the photographs have been broadened with the Clepsydra apparatus.

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NOTE ON POLYTROPIC EQUILIBRIUM IN CURVED SPACE.

G. C. McVittie, Ph.D.

I

Recently Professor E. A. Milne † has indicated the importance of studying the question of polytropic equilibrium under a given external pressure, and has considered the case when this external pressure is due to the photosphere of the star. In the present paper we suppose the external pressure to be produced by an "Einstein universe" distribution of matter outside the star. By this we mean that the external matter is approximately equivalent to a gas of constant density, whose pressure is small and due only to the disturbing effect of the "mass-particle" which is the star. We also

* *Ap. J.*, **69**, 173, 1929; **70**, 85 and 237, 1929; **72**, 277, 1930; **74**, 225, 1931.
† *M.N.*, **96**, 184–218, January 1936.

assume that the external distribution of matter, while not necessarily occupying the whole universe, does occupy a region which is very large compared with the dimensions of the star. Such an external distribution of matter is known * to have the metric

$$ds^2 = \left\{\frac{1-\frac{m}{2r}\left(1+\frac{r^2}{4R^2}\right)^{\frac{1}{2}}}{1+\frac{m}{2r}\left(1+\frac{r^2}{4R^2}\right)^{\frac{1}{2}}}\right\}^2 dt^2 - \frac{\left\{1+\frac{m}{2r}\left(1+\frac{r^2}{4R^2}\right)^{\frac{1}{2}}\right\}^4}{\left(1+\frac{r^2}{4R^2}\right)^2}\frac{1}{c^2}\{dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)\}, \quad (1.1)$$

where m is the total mass (in units of length) of the star and R is a constant defining the density of the external matter. The problem is essentially to fit the metric (1.1) on to the metric defining the field inside the boundary of the star, within which the equilibrium is assumed to be polytropic, of any index n, and describable by the solution without central singularity of the corresponding Emden equation. To do this we assume that there is continuity of gravitational force and of pressure at the boundary of the star, but not necessarily of density. The solution of the problem is an equation connecting the mass, radius and surface density of the star with the density of the external matter, and involving only Emden functions besides. The equation is thus independent of the constant k involved in the polytropic relation $p = k\rho^{1+\frac{1}{n}}$. The value of the external density can thus be found for each value of n. It turns out to be small, but rapidly increasing, between $n = 1$ and $n = 2\frac{1}{2}$. Thereafter its rate of increase is much smaller, but, on the whole, the value of the external density for $n = 3$ is fairly high.

II

(a) *Internal Field.*—We have first to find the metric for the interior of the star where the equilibrium is in the first approximation polytropic. We require only the approximate values of the coefficients of the metric, which we assume to be of the form

$$ds^2 = e^{\zeta(r)}dt^2 - \frac{e^{\nu(r)}}{c^2}\{dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)\}, \quad (2.1)$$

the origin of space co-ordinates being at the centre of the star. Using Einstein's gravitational equations we obtain † the following expressions for the density ρ and the pressure p (a dash denotes differentiation with respect to r):—

$$\frac{\kappa}{c}\rho = -\frac{\lambda}{c^2} - e^{-\nu}\left\{\nu'' + \frac{2}{r}\nu' + \tfrac{1}{4}(\nu')^2\right\} \quad (2.2)$$

$$\frac{\kappa}{c}p = \frac{\lambda}{c^2} + \frac{e^{-\nu}}{4}\left\{\zeta'' + \nu'' + \frac{3}{r}(\zeta'+\nu') + \tfrac{1}{2}(\zeta'+\nu')^2\right\}, \quad (2.3)$$

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* G. C. McVittie, *M.N.*, **93**, 325–339, 1933.

† G. C. McVittie, *loc. cit.*, equations (11), (49), (50).

where λ is a constant which we shall later interpret in terms of the external density and

$$\frac{\kappa}{c}=\frac{8\pi G}{c^4}=2\times 10^{-48}\text{ c.g.s. units.} \tag{2.4}$$

In these formulæ ρ is expressed in ergs/cm.^{3} whilst p, which stands for the sum of the gas and radiation pressures, is in dynes/cm.^{2}. In addition to equations (2.2) and (2.3) we have the equation for the isotropy of pressure, viz. :—

$$\zeta''+\nu''-\frac{1}{r}(\nu'+\zeta')-\nu'\zeta'-\tfrac{1}{2}(\nu')^2+\tfrac{1}{2}(\zeta')^2=0. \tag{2.5}$$

We expand ζ and ν as series in powers of κ/c and neglect terms of higher order than the second. Thus we can put

$$\left.\begin{aligned}\zeta&=-\frac{\kappa}{c}\zeta_1-\left(\frac{\kappa}{c}\right)^2\zeta_2,\\ \nu&=-\frac{\kappa}{c}\nu_1-\left(\frac{\kappa}{c}\right)^2\nu_2.\end{aligned}\right\} \tag{2.6}$$

Introducing these into (2.5) and equating coefficients of successive powers of (κ/c) to zero we obtain

$$\zeta_1''+\nu_1''-\frac{\nu_1'+\zeta_1'}{r}=0, \tag{2.7}$$

$$\zeta_2''+\nu_2''-\frac{\nu_2'+\zeta_2'}{r}+\nu_1'\zeta_1'+\tfrac{1}{2}(\nu_1')^2-\tfrac{1}{2}(\zeta_1')^2=0. \tag{2.8}$$

The solution of (2.7) we require is

$$\zeta_1+\nu_1=0. \tag{2.9}$$

Using this result in (2.8) we obtain without difficulty

$$\zeta_2'+\nu_2'=r\int\frac{(\nu_1')^2}{r}dr+\alpha r, \tag{2.10}$$

where α is a constant of integration. The formulæ (2.2) and (2.3) are now

$$\frac{\kappa}{c}\rho=-\frac{\lambda}{c^2}+\frac{\kappa}{c}\left\{\nu_1''+\frac{2\nu_1'}{r}\right\}+O\left(\frac{\kappa^2}{c^2}\right), \tag{2.11}$$

$$\frac{\kappa}{c}p=\frac{\lambda}{c^2}-\left(\frac{\kappa}{c}\right)^2\left\{\int\frac{(\nu_1')^2}{r}dr+\frac{(\nu_1')^2}{4}+\alpha\right\}. \tag{2.12}$$

Since these formulæ must reduce approximately to the polytropic relation

$$p=k\rho^{1+\frac{1}{n}}, \tag{2.13}$$

where k is a positive constant, we must have

$$\frac{\lambda}{c^2}=\left(\frac{\kappa}{c}\right)^2\alpha,$$

so that λ/c^2 is at most of the second order in κ/c. Thus the equation (2.11) is, to a first approximation,

$$\rho = \nu_1'' + \frac{2\nu_1'}{r}.$$

It follows that, to order κ/c, the functions ν and ζ are given by

$$\nu_1' = -\zeta_1' = \frac{1}{4\pi}\frac{M(r)}{r^2}, \tag{2.14}$$

where $M(r) = 4\pi\int\rho r^2 dr$, and is therefore the mass (in ergs) of the matter inside radius r. The equation (2.12) now reduces to

$$p = -\frac{\kappa}{2c}\int\rho\nu_1' dr,$$

which, on differentiation and introduction of (2.14), yields the well-known Newtonian equation of mechanical equilibrium :

$$\frac{dp}{dr} = -\frac{\kappa}{8\pi c}\frac{M(r)}{r^2}\rho. \tag{2.15}$$

For our purpose the coefficients of the metric (2.1) are determined sufficiently accurately by equation (2.14).

(b) *External Field.*—The exact expressions for the density and pressure in the external field (1.1) are given by

$$\left.\begin{aligned}\frac{\kappa}{c}\rho_e &= -\frac{\lambda}{c^2} + \frac{3}{R^2}\left\{1 + \frac{m}{2r}\left(1 + \frac{r^2}{4R^2}\right)^{\frac{1}{2}}\right\}^{-4} - \frac{3}{R^2}\frac{m}{2r}\left(1 + \frac{r^2}{4R^2}\right)^{\frac{1}{2}}\left\{1 + \frac{m}{2r}\left(1 + \frac{r^2}{4R^2}\right)^{\frac{1}{2}}\right\}^{-5} \\ \frac{\kappa}{c}p_e &= \frac{\lambda}{c^2} - \frac{1}{R^2}\left\{1 + \frac{m}{2r}\left(1 + \frac{r^2}{4R^2}\right)^{\frac{1}{2}}\right\}^{-5}\left\{1 - \frac{m}{2r}\left(1 + \frac{r^2}{4R^2}\right)^{\frac{1}{2}}\right\}^{-1}.\end{aligned}\right\} \tag{2.16}$$

Since the state of affairs consists of a distribution of matter in an approximately "Einstein universe" condition, *i.e.* with constant density and very small pressure, we put *

$$\frac{\lambda}{c^2} = \frac{1}{R^2}. \tag{2.17}$$

The constant $1/R^2$ is thus of the second order in κ/c. Moreover, for any star with mass and radius comparable with those of the Sun it is known that the ratio m/r is very small at the boundary of the star. Thus we can take, instead of (2.16), the approximate expressions (using (2.17)) :

* The relation (2.17) implies only that λ is a "local" cosmical constant, *i.e.* a constant necessary to define the "Einstein" state of the matter in the external density distribution, and not the cosmical constant of the theory of the universe as a whole. This allows us to suppose, for instance, that $\frac{1}{R^2}$ is not a true constant but decreases slowly with r until, at the confines of the external distribution of matter, the metric (1.1) is no longer applicable but gives way to the general metric of the universe.

$$\frac{\kappa}{c}\rho_e=\frac{2\lambda}{c^2}, \qquad \frac{\kappa}{c}p_e=\frac{2m}{r}\frac{\lambda}{c^2}.$$

Hence finally, at the boundary of the star and beyond, we have

$$p_e=\frac{m}{r}\rho_e. \tag{2.18}$$

(c) *Equations of Fit.*—It remains to fit together the fields given by (1.1) and (2.1) respectively. We assume that r denotes the same co-ordinate for both metrics. The continuity conditions we impose at the boundary of the star are (i) that the gravitational force should be continuous, (ii) that the pressure should also be continuous. We denote by the suffix zero the values of all quantities at the boundary of the star. Then condition (i) is equivalent to equating the first derivatives of corresponding coefficients of the metrics (2.1) and (1.1), the equality holding to order κ/c only. Using (2.14) we get

$$\frac{1}{4\pi}\frac{\kappa}{c}\frac{M_0}{r_0^2}=\frac{2m}{r_0^2}.$$

Hence

$$m=\frac{1}{8\pi}\frac{\kappa}{c}M_0, \tag{2.19}$$

and is therefore the mass of the star in centimetres. Again, by (2.18), the continuity of pressure is expressed by

$$p_0=(p_e)_0=\frac{1}{8\pi}\frac{\kappa}{c}\frac{M_0}{r_0}\rho_e, \tag{2.20}$$

in which p_0 is to be determined from the equations (2.13) and (2.15).

III

Polytropic Equilibrium.—We introduce the usual reduction of equations (2.13) and (2.15) by putting

$$\left.\begin{aligned}\rho&=\rho_c\theta^n,\\ r&=A^{\frac{1}{2}}\rho_c^{\frac{1-n}{2n}}\xi,\\ p&=k\rho_c^{\frac{1+n}{n}}\theta^{n+1},\\ M(r)&=4\pi A^{\frac{3}{2}}\rho_c^{\frac{3-n}{2n}}\left(-\xi^2\frac{d\theta}{d\xi}\right),\end{aligned}\right\} \tag{3.1}$$

where $A=k(n+1)2c/\kappa$ and ρ_c is the (finite) central density. The equation (2.15) then becomes

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{d\theta}{d\xi}\right]=-\theta^n,$$

and we consider only that one of its solutions for which, at $\xi=0$, $\theta=1$ and $\frac{d\theta}{d\xi}=0$. The equation (2.20) is now, in virtue of (3.1),

$$\rho_c\theta_0^{n+1} = \left\{ -(n+1)\xi\frac{d\theta}{d\xi}\right\}_0 \rho_e, \tag{3.2}$$

where, as before, a zero denotes the value of any quantity at the boundary of the star. We now introduce the two functions * V and U, which are associated with the Emden functions, where

$$V = -\frac{(n+1)}{\theta}\xi\frac{d\theta}{d\xi}, \qquad U = -\frac{\xi\theta^n}{\frac{d\theta}{d\xi}}. \tag{3.3}$$

If ρ_0 is the density inside the star at its boundary, we have $\rho_0 = \rho_c\theta_0^n$ and therefore the equation (3·2) becomes

$$\rho_0 = \rho_e V_0. \tag{3.4}$$

This therefore is the final form of the equation of continuity of pressure at the boundary of the star and it is independent of the polytropic constant k.

Again, from the second and fourth of equations (3.1) we obtain, by elimination of A, another equation independent of k, viz. :—

$$\frac{M_0}{r_0^3} = 4\pi\rho_c\left(-\frac{1}{\xi}\frac{d\theta}{d\xi}\right)_0 = 4\pi\rho_0\left(-\frac{1}{\xi\theta^n}\frac{d\theta}{d\xi}\right)_0.$$

If we make use of (3.4) and of the definitions of U and V we can put this relation into the form

$$\frac{M_0}{r_0^3} = 4\pi\rho_e\left(\frac{V}{U}\right)_0. \tag{3.5}$$

The equations (3.4) and (3.5) are those which a polytrope of arbitrary index n must satisfy in order that it may be in equilibrium in an extensive "cloud" of matter whose density, ρ_e, is approximately constant and whose pressure is given by (2.18).

Our approximations are, however, only valid if m/r_0 and $1/R^2$ are both small. From (2.18) it follows that p_e must also be small, and hence, since the pressure is continuous at the boundary, p_0 and θ_0 must be small. Thus (3.4) and (3.5) are valid only near the first zero of the Emden function θ, for all n. If $\xi = x_0$ denotes this zero, we know † that for small values of $t = \frac{1}{\xi} - \frac{1}{x_0}$ the function θ is given by

$$\theta = C_0 t - \frac{C_0^n x_0^4 t^{n+2}}{(n+1)(n+2)} - \cdots,$$

where $C_0 = -x_0^2\left\{\frac{d\theta}{d\xi}\right\}_{\xi=x_0}$ and terms of higher order in t are neglected.

Using this series we obtain the approximate values

$$V_0 = \frac{n+1}{\xi_0 t_0}, \qquad U_0 = \xi_0^3 t_0^n C_0^{n-1}.$$

* *B.A. Mathematical Tables*, vol. ii, British Association, 1932, p. iv.

† *B.A. Mathematical Tables, loc. cit.*, p. vi.

Inserting these values into (3.4) and (3.5) and eliminating t_0 we obtain, after rearrangement,

$$\rho_e = \frac{1}{n+1}\rho_0^{\,1+\frac{1}{n}}\left(\frac{\xi_0^{\,n-3}}{C_0^{\,n-1}}\right)^{\frac{1}{n}}\left(\frac{4\pi r_0^{\,3}}{M_0}\right)^{\frac{1}{n}}. \tag{3.6}$$

In this expression for the external density ξ_0 is very nearly equal to x_0 and is therefore approximately known for each n. Similarly C_0 is known. Hence ρ_e is determinate as soon as we know the polytropic index and the mass, radius and surface density of the star.

We consider the variation of ρ_e with n for a given type of star. We suppose the latter to have the mass and radius of the Sun, and put, roughly,

$$r_0 = 7 \times 10^{10} \text{ cm.}, \qquad M_0 = 2 \times 10^{54} \text{ ergs.}$$

It is hardly possible to choose as precise a value for ρ_0. It is known that at the base of the Sun's hydrogen chromosphere the density is of the order of 16×10^{-13} grs./cm.^{3},* whilst for the calcium chromosphere the value is 10^{-17} grs./cm.^{3}.† We adopt the value $\rho_0/c^2 = 10^{-15}$ grs./cm.^{3} for the boundary density and thereby assume that the transition from polytropic to "Einstein universe" equilibrium takes place at densities of chromospheric order. Using the tables of the Emden functions and (3.6) we obtain the following values of ρ_e/c^2 (in grs./cm.^{3}) :—

n	1	1½	2	2½	3	3½	4
ρ_e/c^2	$2{\cdot}2 \times 10^{-31}$	$1{\cdot}3 \times 10^{-26}$	$4{\cdot}9 \times 10^{-24}$	$2{\cdot}2 \times 10^{-22}$	$1{\cdot}6 \times 10^{-21}$	$1{\cdot}3 \times 10^{-20}$	6×10^{-20}

Between $n = 1$ and $n = 2\frac{1}{2}$ it is seen that the density of the external matter rises very rapidly with n, but that between $n = 2\frac{1}{2}$ and $n = 4$ there is, comparatively, only a small increase of ρ_e/c^2. If we imagine the density of the external matter to be capable of change, we should conclude that small values of n imply great stability of the polytropic form, but that for the values of n near $n = 3$ the contrary is the case. We also remark that, even with our low value of ρ_0/c^2, polytropes of $n = 3$ require an unexpectedly high value for the external density. It is to be remembered that if the contents of the whole visible universe were spread out evenly throughout it, the resulting density would only be of order 10^{-30} grs./cm.^{3}. On the other hand, Jeans ‡ has shown that stars of mass similar to that of the Sun would be formed only in those denser parts of a "cosmic cloud" where the density was of order 10^{-21} to 10^{-22} grs./cm.^{3}. In a general way our result would lead us to expect that such stars would be polytropes with indices greater than $2\frac{1}{2}$. It should also be remarked that the polytrope $n = 3$ does not, from the present point of view, possess any peculiar properties as compared with the other polytropes of large index.

The University,
Liverpool :
1936 *April.*

* S. Rosseland, *Astrophysik*, Julius Springer, Berlin, 1931, p. 188.
† A. S. Eddington, *Internal Constitution of the Stars*, C.U.P., 1926, p. 368.
‡ *Astronomy and Cosmogony*, 2nd ed., C.U.P., 1929, p. 349.

REMARKS ON THE IDENTIFICATION OF FORBIDDEN LINES.

J. C. Boyce, Ph.D.

The identification of forbidden lines in the spectra of gaseous nebulæ was summarized more than a year ago by Bowen * and shortly afterwards by Swings.† Recent advances in the analysis of spectra and recent observational material ‡ permit a revision of these earlier tables. Occasion is taken to add remarks on a number of points.

1. New analyses of laboratory spectra have been made by Edlén on *F*II, §*F*III, §*F*IV, ‖ by Robinson on *P*I,¶ *P*II,¶ *S*III,** *K*VI,** by the present writer on *A*III,†† *A*IV.‡‡ Previous argon (III and IV) identifications are confirmed. The line at λ 6311 is shown by Robinson to be the auroral type transition in *S*III. This confirms the earlier tentative identification privately communicated by Bowen which was based on an interpolation from Robinson's data on *P*II. Lines due to the above stages of ionization in *F*, *P* and *K* are definitely absent from available spectra of nebulæ. Edlén's results on *F*II make more exact the extrapolation identifying the nebular pair due to *Ne*III. Results on *A*III revise the interpolated values for unobserved lines of *Cl*II and the extrapolated value for the unobserved auroral line of *K*IV. Robinson's results on *S*III and *K*VI improve the accuracy of the interpolation, confirming the identification of the nebular pair of *A*V.

2. On the observational side Stoy §§ has extended the knowledge of the red end of the spectra of nebulæ and has summarized results over the whole spectral range, combining his own results with the very extensive earlier observations by Wright and with some measurements by Merrill. Stoy lists 96 lines observed with certainty in the spectra of gaseous nebulæ. Of these 58 are permitted lines of *H*, *He*, *C*, *N* and *O*. Thirty lines have now been definitely identified by Bowen and others as due to forbidden transitions and 4 more lines have been similarly identified with varying

* I. S. Bowen, *Ap. J.*, **81**, 1, 1935. (Summary of all work to that date is included in paper.) Professor Bowen has kindly informed me, in commenting on this manuscript, that a new article by him appears in *Reviews of Modern Physics*, 8, No. 2, p. 55 (April 1936), dealing with the more physical aspects of the problem.

† P. Swings, *Les Spectres des Neb. Gaz.*, Hermann et Cie., Paris, 1935 ("Act. Scient. et Indust.," 241 ; "Exp. d'Astr.," Stel. V.).

‡ R. H. Stoy, *L.O.B.* 480, 1935.

§ B. Edlén, *Zs. f. Phys.*, **93**, 433, 1935.

‖ B. Edlén, *Zs. f. Phys.*, **92**, 19, 1934.

¶ H. A. Robinson, *Phys. Rev.*, **49**, 297, 1936.

** H. A. Robinson, *Nature*, **137**, 992, 1936. (I am indebted to Dr. Robinson for his kindness in transmitting his results to me prior to publication.)

†† J. C. Boyce, *Phys. Rev.*, **49**, 351, 1936.

‡‡ J. C. Boyce, *Phys. Rev.*, **48**, 396, 1935.

§§ R. H. Stoy, *L.O.B.* 480, 1935.

TABLE I

Observed Forbidden Lines and Unknown Lines in the Spectra of Gaseous Nebulæ

Spectrum	λ Nebulæ (Wright-Merrill-Stoy)	λ Calculated			Transition	Notes	
		Nebular	Auroral	Transauroral		λ	Δλ
*Ne*III	3343		3342·8 ± 2		$^1D_2 - {}^1S_0$	U1	
*Ne*V	3345·9	3344 ± 20			$^3P_1 - {}^1D_2$	E1	E1
*Ne*V	3425·8	3424 ± 20			$^3P_2 - {}^1D_2$	E1	E1
*O*II	3726·12	3726·2 ± 1			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{1\frac{1}{2}}$	I1	U1
*O*II	3728·91	3729·1 ± 1			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	I1	U1
*Ne*III	3868·74	3855 ± 20			$^3P_2 - {}^1D_2$	E2	U1
*Ne*III	3967·51	3953 ± 20			$^3P_1 - {}^1D_2$	E2	U1
*S*II	4068·62			4068·5 ± 2	$^4S^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	I1	U1
*S*II	4076·22			4076·5 ± 2	$^4S^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	I1	U1
*O*III	4363·21		4363·2 ± 1		$^1D_2 - {}^1S_0$	U1	
?	4571						
*A*IV	4711·4	4711·4 ± 1			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	I3	U3
*Ne*IV?	4725·5		See text		$^2D^\circ_{2\frac{1}{2},1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2},\frac{1}{2}}$	U1	?
*A*IV	4740·2	4740·3 ± 1			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{1\frac{1}{2}}$	I3	U3
*O*III	4958·91	4959·5 ± 2			$^3P_1 - {}^1D_2$	I1	U1
*O*III	5006·84	5007·6 ± 2			$^3P_2 - {}^1D_2$	I1	U1
?	5183 ±						
? (*Ca*V?)	5313 ±	(5308·9 ± 5)			$(^3P_2 - {}^1D_2)$	(I1	U1)
*Cl*III	5519	5517·2 ± 4			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	I1	U1
*Cl*III	5538	5537·7 ± 4			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{1\frac{1}{2}}$	I1	U1
*O*I	5577		5577·341 ± 0·02		$^1D_2 - {}^1S_0$	L1	
?	5720						
*N*II	5754·8		5755·0 ± 1		$^1D_2 - {}^1S_0$	U1	
? (*Ca*V?)	6085	(6085·9 ± 5)			$(^3P_1 - {}^1D_2)$	(I1	U1)
*K*IV?	6102	6101·1 ± 6			$^3P_2 - {}^1D_2$	I1	U1
*O*I	6300·2	6300·23 ± 0·2			$^3P_2 - {}^1D_2$	L1	L1
*S*III	6311		6310·2 ± 1		$^1D_2 - {}^1S_0$	U4	
*O*I	6363·9	6363·88 ± 0·2			$^3P_1 - {}^1D_2$	L1	L1
*A*V	6435	6436·5 ± 5			$^3P_1 - {}^1D_2$	E4	U3 E4
?	6492						
*N*II	6548·1	6548·4 ± 1			$^3P_1 - {}^1D_2$	I1	U1
*N*II	6583·6	6583·9 ± 1			$^3P_2 - {}^1D_2$	I1	U1
*S*II	6717	6717·3 ± 4			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	I1	U1
*S*II	6730	6731·5 ± 4			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{1\frac{1}{2}}$	I1	U1
*A*V	7007	7008 ± 5			$^3P_2 - {}^1D_2$	E4	U3 E4
*A*III	7135·6	7135·7 ± 1			$^3P_2 - {}^1D_2$	I5	
*O*II	7319·9		7318·6 ± 2		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2},1\frac{1}{2}}$	U1	U1
			7319·4 ± 2				
*O*II	7330·4		7329·9 ± 2		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2},1\frac{1}{2}}$	U1	U1
			7330·7 ± 2				

Notes

Nature of data used in calculations—

E Extrapolation or interpolation along iso-electronic sequence.
I Intersystem combination lines (often in extreme ultraviolet).
L Line actually excited in laboratory. U Lines in extreme ultraviolet.

References to data used in calculations—

1. I. S. Bowen, *Ap. J.*, **81**, 1, 1935 (summary of work to that date).
2. Extrapolation improved by use of *F* II data of B. Edlén, *Zs. f. Phys.*, **93**, 433, 1935.
3. J. C. Boyce, *Phys. Rev.*, **48**, 396, 1935.
4. H. A. Robinson, *Nature*, **137**, 992, 1936.
5. J. C. Boyce, *Phys. Rev.*, **49**, 351, 1936.

degrees of uncertainty. Four lines, all faint, remain for which no likely interpretation has been advanced. The elements represented by the forbidden lines are *N*, *O*, *Ne*, *S*, *Cl*, *A*, *K* (?) and *Ca* (??).

3. Predictions and observational material are presented in tabular form. Table I lists, in order of wave-length, the forbidden lines from Stoy's list and the lines of unknown origin. Revised predicted values (with tolerances) are given and some symbols to denote the basis upon which each prediction rests. Except for the spectra mentioned in section 1 all these data are taken directly from Bowen's paper which contains a full bibliography. Table II contains the wave-lengths of all other forbidden lines which might

TABLE II

Forbidden Lines NOT Observed in the Spectra of Gaseous Nebulæ

Spectrum	Nebular	Auroral	Transauroral	Transition	Notes λ	Notes $\Delta\lambda$
F III	2930·0 ± 0·5			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{1\frac{1}{2}}$	I2	U2
F III	2933·1 ± 0·5			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	I2	U2
Ne V		2972 ± 20		$^1D_2 - {}^1S_0$	E1	
O I			2972·31 ± 0·05	$^3P_1 - {}^1S_0$	I1	
N II			3063·0 ± 0·5	$^3P_1 - {}^1S_0$	I1	
Al VII		3068·8 ± 3		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
Al VII		3074·0 ± 3		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
Al VII		3093·4 ± 3		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
Al VII		3098·7 ± 3		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
Cl IV			3118·3 ± 2	$^3P_1 - {}^1S_0$	I1	
Na IV	3245 ± 30			$^3P_2 - {}^1D_2$	E1	U1
A III			3250 ± 0·5	$^3P_1 - {}^1S_0$	I5	
Cl III			3342·7 ± 2	$^4S^\circ_{1\frac{1}{2}} - P^\circ_{1\frac{1}{2}}$	I1	U1
Cl III			3353·4 ± 2	$^4S^\circ_{1\frac{1}{2}} - P^\circ_{\frac{1}{2}}$	I1	U1
Na IV	3366 ± 30			$^3P_1 - {}^1D_2$	E1	U1
N I			3466·4 ± 0·5	$^4S^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2},\,1\frac{1}{2}}$	I1	
Mg VI		3485·5 ± 3		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
Mg VI		3488·1 ± 3		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
Mg VI		3500·4 ± 3		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1

Spectrum	Nebular	Auroral	Transauroral	Transition	Notes λ	Notes Δλ
*Mg*VI		3503·0 ± 3		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*F*IV		3532·2 ± 1		$^1D_2 - {}^1S_0$	U7	
*Ca*VI	3646 ± 200			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	E1	U1
*Ca*VI	3703 ± 200			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{1\frac{1}{2}}$	E1	U1
*S*III			3721·1 ± 1	$^3P_1 - {}^1S_0$	I4	
*Cl*II			3893 ± 40	$^3P_1 - {}^1S_0$	E5	
*Ca*V		3996·3 ± 5		$^1D_2 - {}^1S_0$	U1	
*F*IV	3996·3 ± 1			$^3P_1 - {}^1D_2$	I7	U7
*Na*V		4011·2 ± 3		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
*Na*V		4015·3 ± 3		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
*Na*V		4017·5 ± 3		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*Na*V		4021·6 ± 3		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*F*IV	4059·3 ± 1			$^3P_2 - {}^1D_2$	I7	U7
*K*VI		4097 ± 75		$^1D_2 - {}^1S_0$	E4	
*K*V	4112 ± 100			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	E1	U1
*K*V	4153 ± 100			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{1\frac{1}{2}}$	E1	U1
*F*II		4157·5 ± 1		$^1D_2 - {}^1S_0$	U2	
*S*I			4589·0 ± 1	$^3P_1 - {}^1S_0$	I1	
*A*V		4610 ± 50		$^1D_2 - {}^1S_0$	E4	
*C*I			4621·4 ± 1	$^3P_1 - {}^1S_0$	I1	
*P*II			4669 ± 1	$^3P_1 - {}^1S_0$	I6	
*K*IV		4695 ± 30		$^1D_2 - {}^1S_0$	E5	
*F*II	4789·5 ± 1			$^3P_2 - {}^1D_2$	I2	U2
*F*II	4869·3 ± 1			$^3P_1 - {}^1D_2$	I2	U2
*N*I	5198·5 ± 1			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{1\frac{1}{2}}$	I1	U1
*N*I	5200·7 ± 1			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	I1	U1
*Ca*V	5308·9 ± 5			$^3P_2 - {}^1D_2$	I1	U1
*Cl*IV		5322·2 ± 4		$^1D_2 - {}^1S_0$	U1	
*P*I			5332·4 ± 1	$^4S^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	I6	U6
*P*I			5339·7 ± 1	$^4S^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	I6	U6
*Ca*VI		5460·0 ± 5		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
*Ca*VI		5587·2 ± 5		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
*A*III		5596·6 ± 1		$^1D_2 - {}^1S_0$	U5	
*K*VI	5603 ± 2			$^3P_1 - {}^1D_2$	I4	U4
*Ca*VI		5631·0 ± 5		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*F*III		5721·2 ± 1		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2},\frac{1}{2}}$	U2	U2
*F*III		5733·0 ± 1		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2},\frac{1}{2}}$	U2	U2
*Ca*VI		5766·4 ± 5		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*Ca*V	6085·9 ± 5			$^3P_1 - {}^1D_2$	I1	U1
*K*V		6223·4 ± 5		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
*K*VI	6229 ± 2			$^3P_2 - {}^1D_2$	I4	U4
*K*V		6316·6 ± 5		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
*K*V		6349·5 ± 5		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*K*V		6446·5 ± 5		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*Si*I			6526·88 ± 0·1	$^3P_1 - {}^1S_0$	I1	

Spectrum	Nebular	Auroral	Transauroral	Transition	Notes λ	Notes $\Delta\lambda$
*Cl*III		6695 ± 100		$^1D_2 - {}^1S_0$	E5	
*K*IV	6794·8 ± 6			$^3P_1 - {}^1D_2$	I1	U1
*A*IV		7169 ± 2		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U3	U3
*A*IV		7236 ± 2		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U3	U3
*A*IV		7263 ± 2		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U3	U3
*A*IV		7332 ± 2		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U3	U3
*Cl*IV	7530·9 ± 8			$^3P_1 - {}^1D_2$	I1	U1
*S*I		7724·4 ± 3		$^1D_2 - {}^1S_0$	U1	
*A*III	7751·0 ± 1			$^3P_1 - {}^1D_2$	I5	U5
*P*II		7869 ± 1		$^1D_2 - {}^1S_0$	U6	
*Cl*IV	8046·1 ± 8			$^3P_2 - {}^1D_2$	I1	U1
*Cl*III		8433·6 ± 8		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
*Cl*III		8481·6 ± 8		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{1\frac{1}{2}}$	U1	U1
*Cl*III		8501·8 ± 8		$^2D^\circ_{1\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*Cl*III		8550·5 ± 8		$^2D^\circ_{2\frac{1}{2}} - {}^2P^\circ_{\frac{1}{2}}$	U1	U1
*Cl*II	8587 ± 100			$^3P_2 - {}^1D_2$	E5	U1
*C*I		8727·4 ± 3		$^1D_2 - {}^1S_0$	U1	
*P*I	8788 ± 2			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	I6	U6
*P*I	8799 ± 2			$^4S^\circ_{1\frac{1}{2}} - {}^2D^\circ_{2\frac{1}{2}}$	I6	U6
*S*III	9069 ± 3			$^3P_1 - {}^1D_2$	I4	U4
*Cl*II	9130 ± 100			$^3P_2 - {}^1D_2$	E5	U1
*S*III	9535 ± 3			$^3P_2 - {}^1D_2$	I4	U4
*C*I	9822·9 ± 3			$^3P_1 - {}^1D_2$	U1	U1
*C*I	9849·5 ± 3			$^3P_2 - {}^1D_2$	U1	U1

Notes

Nature of data used in calculations—

E Extrapolation or interpolation along iso-electronic sequence.
I Intersystem combination lines (often in extreme ultraviolet).
U Lines in extreme ultraviolet.

References to data used in calculations—

1. I. S. Bowen, *Ap. J.*, **81**, 1, 1935 (summary of work to that date).
2. B. Edlén, *Z. f. Phys.*, **93**, 433, 1935.
3. J. C. Boyce, *Phys. Rev.*, **48**, 396, 1935.
4. H. A. Robinson, *Nature*, **137**, 992, 1936.
5. J. C. Boyce, *Phys. Rev.*, **49**, 351, 1936 (or interpolation revised upon the basis of these argon results).
6. H. A. Robinson, *Phys. Rev.*, **49**, 297, 1936.
7. B. Edlén, *Z. f. Phys.*, **92**, 19, 1934.

arise in the appropriate wave-length region from any of the elements of lighter weight (up to and including calcium). Again this follows Bowen except where new material is available. Because of the uncertainty of identification, two lines of *Ca*V appear in both tables. Table III may be taken as a summary of and index to the two previous tables. All three tables have been arranged to distinguish between nebular, auroral and

TABLE III

Element Index of Forbidden Lines

Spectrum	Nebular		Auroral	Transauroral
*C*I	9822·9	9849·5	8727·4	4621·4
*N*I	5198·5	5200·7	R	3466·4
NII	**6548·1**	**6583·6**	**5754·8**	3063·0
OI	**6300·2**	**6363·9**	**5577**	2972
OII	**3726·12**	**3728·91**	**7318–30**	V
OIII	**4958·91**	**5006·84**	**4363·2**	V
*F*II	4789·5	4869·3	4157·5	V
*F*III	2930·0	2933·1	5721–33	V
*F*IV	3996·3	4059·3	3532·2	V
NeIII	**3868·74**	**3967·51**	**3343**	V
NeIV	V	V	**4725?**	V
NeV	**3345·9**	**3425·8**	2972	V
*Na*IV	3245	3366	V	V
*Na*V	V	V	4011–21	V
*Mg*VI	V	V	3485–3503	V
*Al*VII	V	V	3068–98	V
*Si*I	R	R	R	6526·88
*P*I	8788	8799	R	5332–39
*P*II	R	R	7869	4669
*S*I	R	R	7724·4	4589·0
SII	**6717**	**6730**	R	**4068·62 4076·22**
SIII	9069	9535	**6311**	3721·1
*Cl*II	8587	9130	6695	3893
ClIII	**5519**	**5538**	8433–8550	3342·7 3353·4
*Cl*IV	7530·9	8046·1	5322·2	3118·3
AIII	**7135·6**	7751·0	5596·6	3250
AIV	**4711·4**	**4740·2**	7169–7332	V
AV	**6435**	**7007**	4610	V
KIV	**6102?**	6794·8	4695	V
*K*V	4112	4153	6223–6446	V
*K*VI	5603	6229	4097	V
CaV	**5308·9?**	**6085·9?**	3996·3	V
*Ca*VI	3646	3703	5460–5766	V

Heavy type indicates lines identified in nebular spectra. Details of these identifications are given in Table I, listed in order of wave-length.

Ordinary type indicates lines not found in nebular spectra. Details of these calculations are given in Table II, listed in order of wave-length.

V denotes a line of wave-length shorter than 2900 A., R a line of wave-length longer than 10,000 A.

transauroral types* of transition, since the nature of the transition is important in interpretation of the excitation conditions.

4. In the spectra of gaseous nebulæ (in contrast with certain stages of novæ) nebular type lines are usually stronger than the auroral lines of the same spectrum. This is due in part to the lower excitation potential of the former lines and also to the "by-pass" effect of the transauroral transition

* J. C. Boyce, D. H. Menzel, C. H. Payne, *Proc. Nat. Acad. Sci.*, **19**, 581, 1933. (Harvard Reprint 93.)

in de-exciting the upper metastable state. Quantitative inferences may only be drawn where the theoretical transition probabilities have been computed.* Generally speaking, where both auroral (or transauroral) and nebular lines would fall in the range of observation, any identification of the former line in the absence of the latter should be regarded with suspicion. The converse is not true. In the absence of nebular type lines (even of other stages of ionization) due to *F*, it is unlikely, as Stoy has pointed out, that λ 5720 in nebulæ is the auroral transition of *F*III.

5. Extensions of observations at both ends of the present range are urgently needed. It will be particularly interesting when observations cover all three types of forbidden transition of a given spectrum. *A*III and *N*II would appear to be the most promising spectra for such an investigation.

6. The nebular lines due to *C*, *Mg*, *Al*, *Si* and *P* lie beyond the range of present observations, but carbon is certainly present in the nebulæ with considerable abundance since its permitted lines are included in Stoy's list. No certain conclusions may be drawn as to the relative abundance of the other four of these elements. What might first seem to be indicated, that *Cl* is more abundant than *P*, must be rejected as unproven. Making due allowance for selective effects in excitation and in observation and neglecting hydrogen (of which the nucleus is an elementary particle), there does seem to be evidence for the greater abundance of elements of even atomic weight.

7. The identification of the auroral transition of *Ne*IV was based on the unusual strength of a band at about that wave-length in the spectrum of Nova Persei 1901, together with the unusual strength in that spectrum of the bands due to *Ne*III and to *Ne*V. It has been pointed out that this forbidden line should have two (or four) components. The only measurements of the pertinent extreme ultraviolet lines of *Ne*IV have been made by the present writer.† The resolving power of the vacuum spectrograph was not sufficient for an unambiguous description of the fine structure of the metastable states. The most reasonable interpretation of the partially resolved lines would suggest forbidden transitions giving rise to lines at λλ 4713·0, 4715·2, 4718·6 and 4720·8. But other interpretations are possible which might give a line fairly close to λ 4725 and another blended with the permitted *He*I line at λ 4712·6. Further investigations in the extreme ultraviolet with greater resolving power are needed to clear up this point.

8. Although rejected by Stoy for a number of reasons, a possibility remains that the faint lines at λ 5313 ± and λ 6085 may be the nebular type pair of *Ca*V. When photographic difficulties in the green can be reduced a more accurate measurement of λ 5313 ± will help to decide this question.

9. Dr. Arthur Beer has very kindly prepared for me a list of forbidden lines identified by various investigators in the spectra of eight novæ. Rejecting earlier identifications no longer consistent with the more accurate data contained in Tables I and II, the remaining lines are listed in Table IV. The *F*III identification must now be regarded as more tentative than when it was first proposed, since it was then supported by a supposed *F*II line,

* E. U. Condon, *Ap. J.*, **79**, 217, 1934.

† J. C. Boyce, *Phys. Rev.*, **46**, 378, 1934.

TABLE IV

Forbidden Lines Observed in the Spectra of Novæ

Spectrum	λ Predicted	Nova	Spectrum	λ Predicted	Nova
*Ne*III	3342·8	2, 5, 8	*O*III	4959·9	1, 2, 4, 5, 6, 7, 8
*Ne*V	3344	8, 2	*O*III	5007·6	1, 2, 4, 5, 6, 7, 8
*Ne*V	3424	8, 2	*Ca*V	5308·9	7
*O*II	3726·2	8	*O*I	5577·341	5, 6, 8
*O*II	3729·1		*F*III?	5721·2	7, 8
*Ne*III	*3868·74	2, 5, 6, 7, 8	*F*III?	5733·0	
*Ne*III	*3967·51	2, 5, 6, 7	*N*II	5755·0	4, 5, 6, 7, 8
*S*II	4068·5	6, 7, 8	*Ca*V	6085·9	7, 8
*S*II	4076·5		*O*I	6300·23	5, 6, 8
*O*III	4363·2	1, 2, 5, 6, 7, 8	*O*I	6363·88	5, 6, 8
*C*I	4621·4	1, 3, 4, 5	*A*V	6436·5	8
*A*IV	4711·4	1, 8	*Si*I??	6526·88	7
*Ne*IV	*4725·5	2, 7, 8	*N*II	6583·9	4
*A*IV	4740·3	8	*S*II	6717·3	5

Notes

* Wave-length from spectra of nebulæ used in place of predicted value :

1, Nova Aurigæ 1891 (10) ; 2, Nova Persei 1901 (8) ; 3, Nova Geminorum 1903 (12) ; 4, Nova Lacertæ 1910 (69) ; 5, Nova Geminorum 1912 (13) ; 6, Nova Aquilæ 1918 (52) ; 7, Nova Pictoris 1925 (11) ; 8, Nova Herculis 1934 (79).

The numbers in brackets refer to Stratton's lists of novæ, giving further details, in *Handbuch d. Astrophysik*, **6**, 254, 1928 ; and **7**, 671, 1936.

Literature references are mainly to be found as follows :—

1, 2, 3, 4. Lockyer, *Sol. Phys. Comm. Publ.*, 1914, p. 27.

5. Wright, *Lick O.P.*, **14**, 38, 1925.

6. Cortie, *M.N.*, **79**, 174, 1919 ; Maury, *Harv. Ann.*, **81**, 179, 1920 ; Paddock, *P.A.S.P.*, **30**, 244, 1918 ; **31**, 47, 1919. Also correction by H. Spencer Jones, *M.N.*, **94**, 821, 1934.

7. H. Spencer Jones, *Cape Ann.*, **10**, part 9, 1931 ; *M.N.*, **94**, 35, 1933 ; **94**, 816, 1934.

8. Stratton and Beer, *M.N.*, **95**, 432, 1935 ; Beer, *M.N.*, **95**, 538, 1935 ; **96**, 238, 1936.

Some additional remarks are to be found in Boyce, Menzel and Payne, *Proc. Nat. Acad. Sci.*, **19**, 581, 1933, but many of the predicted wave-lengths in that paper have since been superseded.

now disallowed. It should be recalled that the *F*III identification was rejected as unlikely for λ 5720 in nebulæ (see section 4 above). Perhaps the line in novæ has the same unknown origin. In the spectrum of Nova Herculis it is included in a broad band where the exact interpretation is difficult.

10. Great breadth of lines and the resulting possibility of blends make quite uncertain many identifications in the spectra of novæ, both of forbidden lines and of permitted lines. Abundance discussions based on them

become extremely speculative. However, certain forbidden lines seem to be favourably placed so as to be free from blends. These lines are worth study as indicators of excitation conditions.

I am very glad to thank Professor F. J. M. Stratton for his kindness in extending to me the hospitality of the Solar Physics Observatory during my sojourn in England.

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1936 *April.*

OBSERVATIONS OF NOVÆ, 1935–1936.

W. H. Steavenson.

Most of the observations which follow were made with the 20½-inch Hindle reflector. Except where otherwise stated, references to the comparison stars used will be found in *M.N.*, **92**, 720, 1932.

Nova Aquilæ (1918)

Observed on six nights. There seems now to be little, if any, variability in the light of this nova. The image is sensibly stellar and no signs of nebulosity are detectable visually.

Date	Mag.	Date	Mag.
1935 June 8	10·8	1935 July 24	10·8
11	10·8	Aug. 13	10·7
29	10·7	Nov. 9	10·7

Nova Aquilæ (1925)

Observed on two nights, 1935 June 8 and August 2. On both occasions the magnitude was estimated at 14·9. There has been no appreciable change since 1934, though the nova is still very considerably brighter than before its outburst in 1925. A chart of the field, with magnitudes of comparison stars, was given in *M.N.*, **95**, 640, 1935.

Nova Aurigæ (1891)

Observed on three nights, 1935 September 22, 30, and October 7. On each occasion the magnitude was estimated to be 14·8, which has been its normal brightness for a great many years, with very slight fluctuations.

Nova Cygni (1876)

Observed on twenty-three nights. The slight variability noted in previous years has been continued, though the range ($0^m.4$) has been rather smaller than usual. The mean brightness has also been somewhat below the average observed during the past few years.

Date	Mag.	Date	Mag.	Date	Mag.
1935 June 8	14·76	1935 July 23	15·10	1935 Aug. 27	14·91
11	15·00	30	14·91	Sept. 2	14·91
14	15·00	Aug. 1	14·96	4	14·91
21	14·91	2	15·00	20	14·81
29	14·96	5	15·10	30	14·95
30	14·96	9	15·15	Oct. 5	15·00
July 7	15·00	20	15·10	29	15·00
20	15·00	22	15·00		

Nova Cygni (1920)

Observed on three nights. On 1935 June 8 and August 2 the magnitude was estimated to be 15·5, and on September 30, 15·4. There has been little, if any, change since 1933, but the nova is still considerably brighter than before its outburst in 1920.

Nova Geminorum (1912)

Observed on one night, 1935 October 7, when the magnitude was estimated to be 14·6. The nova has been of about this brightness for a number of years and does not appear to be variable.

Nova Herculis (1934)

Observed on one hundred and thirty-six nights. Up to the end of 1935 May the 20½-inch was used, but thereafter all the observations were made with the 3-inch finder or with binoculars. The magnitudes of comparison stars brighter than 7·4 were taken from *H.A.*, 50 and 54. Those of stars of 7·4 and fainter were taken from the list given by Mitchell and Wirtanen in *A.J.*, 1038.

Following the announcement by Kuiper of the duplicity of the nova, its image was carefully examined with the reflector on 1935 July 23 and was seen to be elongated. The separation of the components at this time was about $0''.2$. By 1936 April the distance had increased to about one-third of a second, and the components were seen just separated on April 18. Both appeared sensibly stellar, and no nebulosity could be seen in their vicinity.

Apart from its rather unusual light-curve, the most striking feature of the nova during the past few months has been its vivid blue-green colour. This was first noticed on 1935 September 22, but probably originated much earlier, being imperceptible in the small apertures used after the end of

May. The green colour persisted and was still just as intense at the time of the last observation here recorded, 1936 April 16.

U.T.	Mag.	U.T.	Mag.	U.T.	Mag.
1935 Apr. 16·96	10·4	1935 July 24·94	6·7	1935 Sept. 28·98	6·6
17·96	10·8	25·98	6·7	29·84	6·6
19·95	11·0	26·91	6·7	30·82	6·6
23·90	12·1	28·90	6·7	Oct. 4·88	6·6
24·97	12·5	30·96	6·7	5·94	6·6
29·95	13·2	31·97	6·7	6·81	6·6
May 1·93	13·2	Aug. 1·94	6·7	7·99	6·6
4·91	12·9	2·92	6·7	8·83	6·6
5·89	12·9	3·95	6·7	10·79	6·6
6·99	12·7	4·92	6·6	11·81	6·6
7·88	12·6	5·94	6·6	13·78	6·7
9·93	12·5	6·92	6·6	15·96	6·6
16·90	11·0	7·97	6·7	17·83	6·7
17·93	11·2	9·93	6·6	18·94	6·7
18·99	10·7	10·90	6·6	19·85	6·7
19·91	10·6	11·87	6·6	20·77	6·7
20·96	10·3	13·92	6·7	21·77	6·6
21·91	10·5	18·92	6·6	22·80	6·7
22·93	10·3	19·91	6·6	29·75	6·7
25·92	9·6	20·94	6·6	Nov. 1·76	6·6
27·93	9·1	21·87	6·6	3·82	6·8
28·94	9·0	22·86	6·6	4·78	6·7
29·96	8·9	25·87	6·6	5·96	6·7
June 2·93	8·4	27·92	6·6	9·74	6·8
3·95	8·4	28·95	6·6	10·75	6·7
5·94	8·4	Sept. 1·95	6·6	12·74	6·8
7·94	8·2	2·84	6·6	15·74	6·8
8·94	8·1	3·85	6·6	25·75	6·7
10·94	8·1	4·91	6·6	26·74	6·7
11·92	8·1	6·85	6·6	Dec. 1·75	6·8
12·95	8·1	7·97	6·6	2·77	6·8
13·92	8·1	9·94	6·7	8·78	6·8
14·98	8·0	11·89	6·7	15·78	6·8
16·93	7·8	12·83	6·6	16·77	6·7
21·97	7·7	13·85	6·6	31·74	6·8
22·98	7·7	14·85	6·6	1936 Feb. 19·01	6·8
24·95	7·8	15·85	6·7	Mar. 5·00	7·2
28·94	7·2	17·85	6·6	16·04	6·9
29·96	7·5	19·85	6·6	18·99	7·0
30·96	7·4	20·83	6·6	23·99	7·1
July 3·93	7·4	21·85	6·6	24·97	7·2
6·98	7·2	22·90	6·6	Apr. 4·97	7·3
7·94	7·1	23·86	6·6	11·94	7·2
18·95	6·7	24·95	6·6	16·98	7·2
20·00	6·6	25·85	6·6		
23·96	6·6	27·87	6·6		

Nova Lacertæ (1910)

Observed on two nights. On 1935 June 8 the magnitude was estimated to be 14·5 and on August 1, 14·3. Variability, if it really exists, is very

slight, but the nova seems during the past few years to have faded by two- or three-tenths of a magnitude, after a long halt at about 14·1.

Nova Lyræ (1919)

Observed on four nights. This nova is perhaps very slightly variable, and its mean brightness is about $0^m.2$ less than in 1934. A chart and list of comparison stars were given in *M.N.*, **95**, 642, 1935.

Date	Mag.
1935 June 5	15·1
8	15·1
11	15·2
Aug. 2	14·8

Nova Ophiuchi (1848)

Observed on twenty nights. The slight variability observed in former years persists, the range in 1935 amounting to $0^m.6$, about the average for this nova. If there is any regular period in the fluctuations, the observations are not sufficiently numerous to establish it.

Date	Mag.	Date	Mag.	Date	Mag.
1935 June 2	12·5	1935 June 21	12·3	1935 Aug. 2	12·4
3	12·6	29	12·8	10	12·6
5	12·6	30	12·9	22	12·7
8	12·7	July 7	12·6	27	12·6
11	12·7	18	12·7	28	12·6
12	12·6	23	12·5	Sept. 2	12·7
14	12·5	30	12·5		

Nova Persei (1901)

Observed on ninety-two nights. The usual marked variability has been continued, the observed range in 1935–1936 amounting to $1^m.9$. The nova is subject to slow changes of brightness, occupying several months, on which many sudden and short-lived oscillations are superimposed. The mean magnitude was considerably above the average, and a marked maximum occurred towards the end of 1935 December. The nebulous wisp south preceding the nova is still visible on good nights, though it seems to be fainter than it was a few years ago.

Date	Mag.	Date	Mag.	Date	Mag.
1935 July 31	13·42	1935 Aug. 22	13·42	1935 Sept. 9	13·47
Aug. 1	13·41	27	13·47	11	13·42
2	13·30	28	13·47	12	13·36
5	13·56	Sept. 1	13·47	17	13·52
9	13·82	2	13·57	19	13·57
10	13·42	4	13·42	20	13·72
20	13·42	6	13·42	22	13·67
21	13·42	7	13·42	23	13·57

Date	Mag.	Date	Mag.	Date	Mag.
1935 Sept. 24	13·57	1935 Nov. 6	12·69	1936 Feb. 2	12·69
25	13·41	8	12·64	7	12·17
27	13·52	9	12·27	8	12·59
30	13·42	10	12·69	9	12·53
Oct. 2	13·00	12	12·69	11	12·48
4	13·15	13	12·74	12	12·59
5	12·74	15	12·72	13	12·53
7	12·90	23	12·92	19	12·32
8	12·84	24	12·87	20	12·37
10	12·69	Dec. 3	13·47	21	12·59
11	12·79	5	13·36	22	12·53
12	12·79	15	12·69	23	12·27
13	13·00	16	12·59	24	12·48
15	13·36	21	12·17	27	12·32
18	12·64	22	11·91	Mar. 5	12·43
20	12·59	27	12·38	8	12·43
22	12·74	29	12·48	18	12·74
25	12·95	31	12·37	23	12·74
29	13·00	1936 Jan. 4	12·54	24	12·79
Nov. 1	12·95	21	12·92	Apr. 4	12·74
3	12·79	22	12·82	11	12·54
4	12·74	26	12·37	26	12·54
5	12·48	27	12·32		

Nova Vulpeculæ (1670)

Some particulars of a faint variable star, found near the reputed position of this object in 1934 July, were given in *M.N.*, **95**, 78, 1934. It is difficult to find out much about the recent history of this variable, as it is too faint to appear on the plates of the Harvard sky-patrol and others taken with small lenses. Its image has, however, been found by Miss Boyd on six plates taken with the 16-inch Metcalf lens at Harvard between 1910 and 1928. Dr. Shapley has sent to me the results of Miss Boyd's examination of these plates, and I give them below by his kind permission. The star 19 has a visual magnitude of about 15·2, while I have estimated 17 at 14·2.

Metcalf Plate No.	Date	Remarks
586	1910 Sept. 22	Doubtfully visible. 19 not shown.
17801	1921 Aug. 23	Brighter than 19.
17821	1921 Aug. 25	Brighter than 19.
22016	1926 July 16	Fainter than 19.
22850	1927 Oct. 29	Brighter than 19.
23687	1928 Oct. 4	Nearly as bright as 17. 19 not shown.

The variability of the small star is thus confirmed, and Miss Boyd further noted that the image was different in character from that of stars of similar magnitude. This peculiarity had already been noted on examination of plates taken at the Radcliffe Observatory in 1934 (*M.N.*, *loc. cit.*).

On a plate taken with the 15-inch doublet of Dr. Wilfred Hall's observatory on 1925 September 21 the star appears about $0^{m}\cdot2$ fainter than

19, while the two objects appear about equal on a plate taken by M. de Kerolyr with the 32-inch mirror on 1935 March 8, and on one taken with the 100-inch Hooker telescope by Dr. van Maanen in 1935 March 26. The following visual estimates were made by myself in 1935 :—

Date	Mag.
1935 June 5	15·4
8	15·4
23	15·5
Aug. 2	15·5

As I have never seen this object brighter than 19, I think it probable that its colour-index is slightly negative, though the discrepancy may, of course, be simply the result of the real variability which undoubtedly exists.

PHOTOMETRIC OBSERVATIONS OF NOVA PERSEI 1901.

B. M. Peek, M.A.

The following visual estimates of the magnitude of Nova Persei, 1901, are a continuation of the series last published in *M.N.*, **93**, 655, 1933, the period covered being from 1933 September 10 to 1936 April 18.

There has been no revision of the magnitudes of the comparison stars since the date of the last publication.

J.D.	Mag.	J.D.	Mag.	J.D.	Mag.
[2427]		[2427]		[2427]	
326·37 (1933)	13·42	439·25 (1934)	13·77	703·36 (1934)	11·97
330·43	13·72	446·42	13·36	707·44	11·97
331·36	13·67	452·25	13·20	709·37	11·97
332·37	13·67	453·26	13·36	710·48	11·97
335·44	13·77	457·45	13·32	715·38	12·32
337·34	13·61	458·41	13·30	716·36	12·54
338·40	13·6 ±	460·33	13·0 ±	722·43	12·22
354·31	13·6 ±	461·25	13·36	733·33	12·7 ±
357·33	13·77	477·33	13·00	734·49	12·6 ±
358·47	13·82	487·31	< 13·3	738·41	12·79
361·44	13·75	488·47	13·47	739·35	12·97
362·38	13·74	500·30	13·30	740·39	12·91
363·47	13·77	504·31	13·25	741·47	12·95
365·43	13·82	515·32	13·04	742·43	13·03
366·27	13·74	...	...	743·44	13·15
371·41	13·54	693·40	12·53	745·39	12·97
390·46	13·59	694·37	12·69	748·49	12·95
411·23	13·55	695·42	12·54	749·48	12·91
414·20	13·84	697·36	12·48	764·25	< 12·7
416·21	13·72	698·40	12·32	781·42	13·25
420·21	13·60	699·38	12·51	784·34	13·30
425·26	13·57	702·46	12·12	798·34	13·15

J.D.	Mag.	J.D.	Mag.	J.D.	Mag.
[2427]		[2428]		[2428]	
801·33 (1934)	13·30	095·29 (1935)	12·87	185·24 (1936)	13·32
811·49 (1935)	13·41	096·41	12·87	186·38	13·07
824·26	13·52	098·47	12·92	190·48	12·87
830·36	12·92	100·44	13·07	194·25	12·51
846·30	12·79	105·47	13·10	195·36	12·64
848·46	12·89	108·48	13·01	197·48	12·69
860·21	13·05	109·31	13·10	204·47	12·32
865·44	12·95	112·47	12·96	206·32	12·48
866·32	12·94	113·32	12·92	207·29	12·74
869·41	12·98	119·33	12·98	208·31	12·87
874·35	12·87	120·38	12·97	209·38	12·77
883·32	12·98	122·44	13·13	210·37	12·76
884·32	13·01	123·33	13·32	211·45	12·82
885·37	13·00	130·30	13·00	212·46	12·74
887·29	12·97	131·31	12·98	218·45	12·48
889·38	12·95	132·29	13·18	220·38	12·76
897·36	12·92	135·41	13·25	222·49	12·71
898·36	12·86	136·36	13·18	226·32	12·73
...	...	138·23	13·25	233·25	12·74
[2428]		139·26	13·36	244·35	12·97
063·46	13·5 ±	140·36	13·54	248·33	12·95
065·38	13·60	152·29	12·98	255·38	12·85
066·37	13·62	153·29	12·95	256·34	13·00
071·37	13·36	158·30	12·40	258·36	12·98
076·34	13·34	159·26	11·7 ±	263·37	12·87
082·43	12·95	160·27	11·7 ±	268·38	12·9 ±
083·30	12·98	163·29	11·9 ±	269·38	12·76
086·49	12·86	164·24	12·0 ±	270·36	12·48
087·45	12·89	168·24	12·0 ±	272·36	12·42
092·44	13·2 ±	181·25 (1936)	13·30	277·37	12·79

THE VELOCITIES OF METEOR STREAMS.

R. A. McIntosh.

Increasing attention is being paid to the determination of the radiants of meteor showers by the telescopic examination of radiant areas, using low powers and wide fields. There is no doubt that such observations should provide very reliable information on the shapes, areas and positions of the radiants of the rich annual showers, provided meteors can be observed in sufficient numbers. Recent experience in America,* Antarctica,† Russia ‡ and New Zealand § shows that with the correct equipment profitable results can be obtained.

With the accurate telescopic determination of meteor radiants a new method of attack on the problem of meteor velocities becomes possible,

* *Pop. Ast.*, **43**, 239, 1935 April.

† Radiogram from Dr. Poulter, Byrd's Second Antarctic Expedition.

‡ *Astronomical Journal of Soviet Union*, **12**, 1, 60, 1935.

§ *Southern Stars* (N.Z. Astronomical Society), **1**, 14, 221, 1935 December.

requiring only that the apparent meteor radiant be determined at two stations as far apart as possible. The writer first conceived the idea in 1929, and it was mentioned at the time in correspondence with Dr. C. P. Olivier, president of the American Meteor Society.

Owing to the inherent errors in naked-eye determinations of meteor radiants, the method then appeared impossible of accomplishment, except in the cases where the meteor radiants were at great elongations from the apex of the Earth's motion. With the recent successful application of telescopes to the observation of meteors this new method of solving meteor velocities becomes practicable.

The displacement of a radiant toward the zenith depends on the zenith distance of the radiant and the velocity of the constituent meteors. It is simple to reverse the usual procedure, determining the meteor velocities from the observed displacement between two radiants determined at stations widely separated in latitude.

The sporadic meteors coming from outside space with highly hyperbolic velocities, discovered by the Harvard Meteor Expedition,* could hardly have their velocities determined by this method, even if they did provide radiants, because the high velocities would result in very small zenith displacements.

There are many annual meteor showers, however, which have not been shown to be related to any known comet. The periods of these meteor streams are therefore not known. Dual telescopic examination of the Delta Aquarid radiant (July) from the northern and southern hemispheres would reveal, for example, whether the meteors travel in an ellipse, parabola or hyperbola.

In the case of the Eta Aquarids (May) we already know the period of the meteor stream because it is undoubtedly related to Halley's Comet. But Dr. H. Svoboda † has suggested that the Orionids (October) are a second manifestation of the Halley Comet meteors near the other node. Dual observations of Orionid radiants would reveal the velocity of the Orionids, and hence the orbit of the stream.

The formula for determining the amount of zenith displacement undergone by a meteor radiant is

$$\tan \tfrac{1}{2}\Delta Z = \frac{Vp - V}{Vp + V} \tan \tfrac{1}{2}Z, \qquad (1)$$

where V is the unperturbed geocentric velocity of the meteors; Vp the same velocity perturbed by the Earth's attraction; Z the zenith distance of the apparent radiant; and ΔZ the displacement of the radiant toward the zenith.

Taking Z_N, Z_S, ΔZ_N, ΔZ_S as the values of Z and ΔZ from northern and southern stations respectively, we have

$$\tan \tfrac{1}{2}\Delta Z_N = \frac{Vp - V}{Vp + V} \tan \tfrac{1}{2}Z_N, \qquad (2)$$

* *Harvard College Observatory Circular*, 389.

† *Bulletin International de l'Académie des Sciences de Bohème*, 1914.

and

$$\tan \tfrac{1}{2}\Delta Z_S = \frac{Vp - V}{Vp + V} \tan \tfrac{1}{2} Z_S. \qquad (3)$$

The fraction $\frac{Vp - V}{Vp + V}$, which is constant for any one meteor stream, naturally has the same value for both northern and southern stations. The angle π (Table I) is the value of the displacement toward the zenith of a radiant on the horizon, where obviously

$$\tan \tfrac{1}{2}\pi = \frac{Vp - V}{Vp + V}. \qquad (4)$$

Since by definition π is the apparent displacement of a radiant on the horizon, Table I is similar to the usual type of table showing the value

TABLE

Combined Zenith

π	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1	0 0	0 3	0 5	0 8	0 11	0 14	0 16	0 19	0 22	0 24	0 28	0 30
2	0 0	0 5	0 10	0 16	0 21	0 26	0 32	0 38	0 44	0 49	0 56	1 2
3	0 0	0 7	0 16	0 23	0 32	0 40	0 48	0 57	1 4	1 14	1 24	1 34
4	0 0	0 10	0 21	0 32	0 44	0 53	1 4	1 15	1 26	1 38	1 52	2 4
5	0 0	0 13	0 26	0 39	0 53	1 3	1 20	1 34	1 49	2 4	2 20	2 36
6	0 0	0 16	0 32	0 47	1 3	1 20	1 37	1 54	2 11	2 29	2 48	3 8
7	0 0	0 18	0 36	0 55	1 14	1 33	1 53	2 12	2 32	2 54	3 16	3 39
8	0 0	0 21	0 42	1 4	1 25	1 47	2 8	2 32	2 54	3 20	3 44	4 10
9	0 0	0 24	0 47	1 11	1 35	2 0	2 25	2 50	3 17	3 44	4 12	4 42
10	0 0	0 26	0 53	1 19	1 46	2 14	2 41	3 10	3 39	4 9	4 40	5 13
11	0 0	0 29	0 58	1 27	1 56	2 26	2 57	3 29	4 1	4 34	5 10	5 44
12	0 0	0 32	1 4	1 35	2 8	2 40	3 14	3 44	4 23	4 56	5 36	6 16
13	0 0	0 34	1 8	1 42	2 18	2 54	3 30	4 7	4 46	5 24	6 5	6 48
14	0 0	0 36	1 14	1 50	2 28	3 6	3 46	4 26	5 7	5 49	6 33	7 19
15	0 0	0 39	1 19	1 59	2 39	3 20	4 2	4 46	5 30	6 14	7 2	7 51
16	0 0	0 42	1 24	2 8	2 50	3 34	4 18	5 4	5 51	6 40	7 30	8 22
17	0 0	0 45	1 30	2 16	3 0	3 48	4 36	5 24	6 14	7 5	7 59	8 54
18	0 0	0 47	1 35	2 24	3 12	4 0	4 52	5 43	6 36	7 30	8 27	9 26
19	0 0	0 50	1 40	2 32	3 22	4 14	5 8	6 3	6 58	7 56	8 56	9 58
20	0 0	0 52	1 46	2 40	3 35	4 28	5 24	6 22	7 21	8 21	9 24	10 30
21	0 0	0 54	1 50	2 48	3 46	4 42	5 40	6 41	7 43	8 47	9 52	11 2
22	0 0	0 58	1 56	2 56	3 56	4 56	5 57	7 1	8 5	9 12	10 22	11 33
23	0 0	1 0	2 2	3 4	4 6	5 10	6 14	7 21	8 28	9 38	10 50	12 6
24	0 0	1 3	2 8	3 12	4 17	5 24	6 31	7 40	8 51	10 4	11 20	12 38
25	0 0	1 6	2 14	3 20	4 28	5 37	6 48	8 0	9 14	10 30	11 48	13 10

of zenith attraction for various zenith distances of the radiant, with the sole difference that the argument is Δ_Z (the zenith attraction at the horizon) instead of ϵ (elongation of the apparent radiant from the apex).

These tables are designed to facilitate the determination of meteor velocities when two apparent radiants of a meteor stream have been observed on the same date. The sum of the zenith distances of the apparent radiants at the two stations is determined. Entering Table I under the appropriate column of combined zenith distances, the value of the observed displace-

ment (the difference in declination if the two radiants were observed on or near the meridian) is found in the body of the table, and the angle π is then read off at the left-hand side. This is the value of the displacement of the apparent radiant toward the zenith when the radiant is on the horizon.

Turning to Table II, the value of π reveals the unperturbed geocentric velocity of the meteor stream in miles per second. The heliocentric velocity can then be determined from the formula

$$M^2 = E^2 + V^2 - 2VE \cos \epsilon, \tag{5}$$

where M is the heliocentric velocity of the meteors, V the unperturbed geocentric velocity found in Table II, E the Earth's orbital velocity and ϵ the apparent elongation of the meteor stream from the meteoric apex (the point toward which the Earth's motion is directed).

I

Distances

π	60°	65°	70°	75°	80°	85°	90°	95°	100°	105°	110°
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1	0 34	0 38	0 42	0 46	0 50	0 55	1 0	1 4	1 12	1 19	1 26
2	1 9	1 16	1 24	1 32	1 40	1 50	2 0	2 11	2 23	2 36	2 52
3	1 44	1 54	2 6	2 18	2 31	2 45	3 0	3 16	3 34	3 54	4 16
4	2 18	2 32	2 48	3 4	3 20	3 40	4 0	4 22	4 46	5 13	5 42
5	2 53	3 11	3 30	3 50	4 11	4 35	5 0	5 28	5 58	6 31	7 8
6	3 28	3 50	4 12	4 36	5 2	5 30	6 0	6 33	7 9	7 49	8 34
7	4 2	4 28	4 54	5 22	5 52	6 25	7 0	7 38	8 20	9 7	10 0
8	4 38	5 6	5 36	6 9	6 43	7 20	8 0	8 44	9 32	10 25	11 24
9	5 12	5 44	6 19	6 55	7 33	8 15	9 0	9 49	10 43	11 43	12 50
10	5 47	6 23	7 1	7 41	8 24	9 10	10 0	10 56	11 54	13 0	14 14
11	6 22	7 1	7 43	8 27	9 14	10 6	11 0	12 0	13 6	14 18	15 40
12	6 57	7 39	8 25	9 13	10 5	11 0	12 0	13 5	14 17	15 36	17 4
13	7 32	8 18	9 8	10 0	10 56	11 56	13 0	14 11	15 28	16 54	18 29
14	8 6	8 57	9 50	10 46	11 46	12 50	14 0	15 16	16 39	18 11	19 54
15	8 42	9 36	10 32	11 32	12 36	13 46	15 0	16 21	17 50	19 28	21 18
16	9 17	10 14	11 14	12 18	13 26	14 40	16 0	17 26	19 1	20 46	22 42
17	9 52	10 52	11 56	13 5	14 18	15 36	17 0	18 32	20 12	22 3	24 6
18	10 27	11 32	12 40	13 52	15 8	16 32	18 0	19 37	21 22	23 20	25 30
19	11 2	12 10	13 22	14 38	15 59	17 26	19 0	20 42	22 34	24 36	26 53
20	11 38	12 48	14 4	15 24	16 50	18 22	20 0	21 47	23 44	25 53	28 16
21	12 13	13 28	14 48	16 12	17 41	19 18	21 0	22 52	24 55	27 10	29 40
22	12 48	14 8	15 30	16 58	18 32	20 12	22 0	23 57	26 6	28 26	31 2
23	13 24	14 46	16 14	17 45	19 22	21 8	23 0	25 2	27 16	29 42	32 24
24	14 0	15 26	16 56	18 32	20 14	22 3	24 0	26 7	28 26	30 58	33 46
25	14 36	16 4	17 38	19 19	21 4	22 58	25 0	27 12	29 36	32 14	35 8

The major axis of the orbit of the meteor stream is found from

$$a = \tfrac{1}{2}\left(\frac{U^2}{U^2 - M^2}\right),$$

where U is the parabolic velocity (26 miles per second) and M the heliocentric velocity of the meteors. If M is less than U the orbit will be an ellipse; if M is greater than U it will be a hyperbola; while if M and U are equal the orbit will be a parabola. Once the velocity and major axis have been

TABLE II

Unperturbed Geocentric Velocity of Meteors

(unit = 1 mile per sec.)

π	V	π	V	π	V	π	V
°		°		°		°	
25·0	5·75	15·5	8·19	6·0	14·46	1·8	27·37
24·5	5·87	15·0	8·36	5·5	15·17	1·7	28·30
24·0	5·99	14·5	8·55	5·0	15·96	1·6	29·22
23·5	6·10	14·0	8·75	4·8	16·33	1·5	30·33
23·0	6·20	13·5	8·95	4·6	16·74	1·4	31·44
22·5	6·29	13·0	9·17	4·4	17·16	1·3	32·55
22·0	6·39	12·5	9·40	4·2	17·59	1·2	33·85
21·5	6·51	12·0	9·64	4·0	18·07	1·1	35·33
21·0	6·62	11·5	9·90	3·8	18·50	1·0	36·99
20·5	6·73	11·0	10·17	3·6	19·05	0·95	38·10
20·0	6·86	10·5	10·47	3·4	19·61	0·90	39·21
19·5	6·99	10·0	10·78	3·2	20·35	0·85	40·32
19·0	7·12	9·5	11·12	3·0	21·09	0·80	41·62
18·5	7·27	9·0	11·47	2·8	21·85	0·75	43·10
18·0	7·40	8·5	11·87	2·6	22·75	0·70	44·58
17·5	7·53	8·0	12·32	2·4	23·68	0·65	46·24
17·0	7·68	7·5	12·78	2·2	24·60	0·60	48·09
16·5	7·84	7·0	13·28	2·0	25·89	0·55	50·68
16·0	8·01	6·5	13·84	1·9	26·63	0·50	53·27

computed the remaining elements of the orbit can be computed by the usual methods.

To illustrate the method I have taken at random two radiants of the Delta Aquarid meteor shower observed in northern and southern latitudes about the same longitude of the meteoric apex, as follows :—

Date, U.T.	L.	R.A.	Dec.	Z.	
	°	°	°	°	
1913 July 30.45	37·0	341·5	−14·6	62·6	*A.M.S.* 193 *
1930 July 29.68	36·7	341·7	−15·7	21·3	*N.Z.* 447 †
			1·1	83·9	

For strict comparison only those radiants observed on the same date and in the same year should be compared. To simplify the calculation I have assumed that both radiants were determined on the observer's meridian, an assumption which is approximately correct. The positions of the stations were taken as: American, $+8^h\ 0^m$, 48° N.; New Zealand, $-11^h\ 36^m$, −36° 50′ S.

Reference to Table I, under $Z = 83°{\cdot}9$, reveals that a displacement of 1°·1 makes the value of $\pi = 1°{\cdot}22$. The value of $\pi = 1°{\cdot}22$ in Table II shows an unperturbed geocentric velocity for these meteors of 33·59 miles per second. Taking $\epsilon = 59°\ 48'$, equation (5) gives a heliocentric velocity of 29·07 miles per second.

* *Publications Leander McCormick Observatory*, 2, part 4, 462.

† *Bulletin* 21, N.Z. Astronomical Society, p. 448.

Since M is greater than U the orbit of the stream is shown to be a hyperbola. I make no claim, however, that this is the true state of affairs. Various selections of naked-eye radiants of this stream will yield widely differing results, ranging from hyperbolas to ellipses of small eccentricity. I do believe, however, that with accurate radiant positions obtained either telescopically or photographically, where the error in determination should not exceed a tenth of a degree, it should be possible to determine reliable orbits for many of the annual showers not related to known comets.

Although the method was formulated by me six years ago, it was not then proceeded with, as no practical use could then be discerned. In the meantime I have been anticipated in publication by the late Dr. Malzev, of Tashkent Observatory,* who advocated its use in the naked-eye study of meteor streams at great elongations from the meteoric apex. The tables in this paper are similar in form to Malzev's, but have been recomputed to allow for the greater accuracy now possible in radiant determinations.

Private Observatory, Epsom,
Auckland, S.E. 3, New Zealand :
1935 *December* 28.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE UNIVERSITY OF LONDON OBSERVATORY BETWEEN 1934 OCTOBER 2 AND 1935 DECEMBER 31.

(Communicated by the Director)

This report concerning occultations observed and reduced at Mill Hill is the fifth of a series of which No. 4 is in *M.N.*, 95, 188, 1934. "Eye and ear" is the method of observation always adopted by G, while F. R., gaining experience, relies less completely on the stop-watch and tends to use it as a valuable confirmation of his "eye and ear" observations.

Examination showed that the average times obtained with the stop-watch were $0^s.51 \pm 0^s.13$ greater than those obtained by eye and ear in the case of D and $0^s.33 \pm 0^s.35$ greater in the case of R. All the times given are those for eye and ear observations. Nos. 97, 99 and 108 were observed by G, the remainder by F. R.

The three co-ordinates of the 24-inch Wilson reflector are given year by year in the *N.A.*, and, since the 8-inch Cooke refractor is no more than 17 metres distant, the same co-ordinates are employed for all the observations. The reflector was used as a Cassegrain of focal length 532 inches with magnification 230, while powers of at first 80, but latterly 130, were used on the refractor of focal length 126 inches.

The error of the Synchronome clock at the instant of an occultation is obtained from two to six straddles of the 6-dot time-signals distributed by the B.B.C.

* *Publications Tashkent Astronomical Observatory*, 4, 2, 3, 1933.

TABLE I

Serial No.	Date	Star	Mag.	Phase	U.T.	χ	$\cos(\chi-\rho)$	$\sin(\chi-\rho)$	$\sigma'-\sigma$
	1934				h m s				
82	Oct. 24	Alcyone	2·9	D	20 58 17·96	284 38	−·901	−·434	+ 1·3
83	24	27 Tau	3·7	D	21 45 20·24	245 23	−·971	+·237	+ 3·4
84	24	28 Tau	5·2	R	21 48 42·68	262 27	−·998	−·058	+ 1·3
85	26	125 Tau	5·1	R	20 34 21·67	246 20	−·967	+·257	[+ 5·4]
86	28	ϵ Gem	3·2	D	0 19 53·53	24 40	+·284	−·959	+ 2·5
87	28	ϵ Gem	3·2	R	0 46 17·00	341 33	−·450	−·893	+ 0·4
88	31	81 Cnc	6·4	R	1 17 51·67	319 5	−·889	−·458	+ 1·3
89	Nov. 18	101 Psc	6·2	D	19 9 45·99	53 47	+·971	−·240	+ 0·4
	1935								
90	Jan. 12	47 B Ari	6·5	D	23 22 1·23	81 30	+·981	+·192	− 2·2
91	16	125 Tau	5·0	D	18 48 13·28	133 53	+·739	+·674	− 0·4
92	Mar. 12	W.Z.C. 414	7·0	D	18 33 0·36	134 4	+·663	+·748	+ 1·4
93	May 26	W.Z.C. 1530	6·6	R	2 54 18·68	286 40	−·750	−·661	+ 2·1
94	June 22	W.Z.C. 1515	7·0	R	1 33 32·83	239 59	−·997	+·079	[+ 8·2]
95	July 18	ρ Aqu	5·4	R	22 37 55·16	216 55	−·882	+·472	[+ 7·0]
96	19	170 B Aqu	6·1	D	23 35 26·66	52 42	+·977	−·213	+ 3·4
97	24	B.D. 19° 432	7·0	R	21 24 44·66	170 58	−·102	+·995	− 1·0
98	24	B.D. 19° 432	7·0	R	21 24 46·66	171 1	−·103	+·995	− 0·7
99	25	B.D. 23° 563	6·1	R	1 24 55·10	292 60	−·845	−·535	+ 2·2
100	Aug. 18	101 Psc	6·2	R	22 25 2·06	263 57	−·965	−·261	[+12·5]
101	21	Hammond 518	6·7	R	2 2 8·18	279 11	−·939	−·343	+ 3·5
102	23	118 Tau	5·4	R	0 49 15·84	276 37	−·991	−·132	[+26·5]
103	Sept. 7	λ Sag	2·9	D	19 29 18·87	47 16	+·814	−·580	− 1·6
104	7	C.D. −25° 13170	6·2	D	20 31 38·62	18 12	+·434	−·901	− 2·0
105	7	λ Sag	2·9	R	20 34 38·14	289 9	−·893	−·449	[+ 8·2]
106	15	20 H′ Ari	6·4	R	21 26 39·49	226 49	−·910	+·414	+ 1·7
107	21	44 Gem	5·9	R	0 45 19·19	307 42	−·660	−·751	+ 1·2
108	21	44 Gem	5·9	R	0 45 22·00	307 40	−·660	−·751	+ 2·0
109	23	34 Cnc	6·3	R	4 3 26·41	278 29	−·980	+·201	+ 1·4
110	Nov. 6	22 B Psc	6·5	D	18 37 12·68	49 27	+·964	−·265	− 1·5
111	6	9 Psc	6·4	D	21 25 59·36	5 39	+·513	−·859	− 1·2
112	13	8 Gem	6·1	R	20 8 53·39	339 2	−·452	−·892	[+11·4]
113	13	9 Gem	6·3	R	20 43 20·70	273 54	−·999	+·041	[+23·9]
114	Dec. 1	18 Aqu	5·5	D	18 40 10·17	81 49	+·971	+·240	− 1·9

TABLE II

Serial No.	Star	R.A. 1935·0	Reduction to Date	Dec. 1935·0	Reduction to Date	Authority
		h m s	s	° ′ ″	″	
92	W.Z.C. 414	6 0 11·05	+2·09	+25 26 52·9	+ 5·7	Mean of Greenwic 1910 and W.Z.C.
93	W.Z.C. 1530	22 53 45·40	+1·96	− 3 35 36·0	+ 9·9	W.Z.C. (wt. 2) an Radcliffe 1890.
94	Hammond 3307	22 38 43·16	+2·93	− 5 26 29·1	+15·4	N.A.O. advance cop Hammond.
97–98	B.D. +19° 432	2 50 35·66	+2·63	+20 18 6·0	+12·4	,, ,,
99	B.D. +23° 563	3 45 51·78	+2·47	+23 30 54·9	+ 9·3	,, ,,
101	W.Z.C. 207	3 33 8·28	+3·42	+22 59 50·6	+13·2	,, ,,
104	C.D. −25° 13170	18 25 25·73	+4·32	−25 17 59·0	− 1·6	Mean of 3 catalogue Cordoba 190 Albany Zone 190 Wilhelmshave 1920.

Column 6 of Table I contains the actual U.T. of the observations. For the purpose of interpolating for parallax, right ascension and declination (but for no other purpose) $0^h.00253$ was added to the U.T. to correct for the known error in the mean longitude as observed by Professor Brown. No other correction for lag, personal errors, contour irregularities on the limb of the Moon, etc., has been included.

The reductions were made by F. R. and checked by G.

The observation of every occultation predicted in the *N.A.* has been attempted—abnormal results being placed in brackets.

F. R. = Frank Robbins.
G = C. C. L. Gregory.

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Corrigendum

M.N., **96**, 390, Presidential Address.—In the reference to Baade's paper on the distance of the Cygnus cloud (*Mount Wilson Contributions*, **21**, No. 493), it should be pointed out that the "cloud" dealt with in the paper is a "star cloud" and not an "obscuration" as stated in the Address.

J. H. R.

Errata

M.N., **96**, 451 (C. R. Burch, On Reflection Compensators), equations (65), *for* $\rho k^2(k+1) = 2F$ *read* $\rho k^2(k+1)^2 = 2F$. P. 453, equations (66), *for* $\rho k^2(k+1) = F$ *read* $\rho k^2(k+1)^2 = F$.